

# Composing with Pitch-Class Sets

# Using Pitch-Class Sets as a Compositional Tool



- Pitches are labeled with numbers, which are enharmonically equivalent (e.g., pc 6 = G flat, F sharp, A double-flat, or E double-sharp), thus allowing for tonal neutrality.
- Reducing pitches to number sets allows you to explore pitch relationships not readily apparent through musical notation or even by ear: i.e., similarities to other sets, intervallic content, internal symmetries.
- The ability to generate an entire work from specific pc sets increases the potential for creating an organically unified composition.
- Because specific pitch orderings, transpositions, and permutations are not dictated as in dodecaphonic music, this technique is not as rigid or restrictive as serialism.
- As with integral serialism, pc numbers may be easily applied to other musical parameters: e.g., rhythms, dynamics, phrase structure, sectional divisions.

# Two Representations of the Matrix for Schönberg's *Variations for Orchestra*

## a. Traditional method, using pitch names

	I <sub>0</sub>	I <sub>6</sub>	I <sub>8</sub>	I <sub>5</sub>	I <sub>7</sub>	I <sub>11</sub>	I <sub>4</sub>	I <sub>3</sub>	I <sub>9</sub>	I <sub>10</sub>	I <sub>1</sub>	I <sub>2</sub>	
P <sub>0</sub>	B $\flat$	E	G $\flat$	E $\flat$	F	A	D	D $\flat$	G	A $\flat$	B	C	R <sub>0</sub>
P <sub>6</sub>	E	B $\flat$	C	A	B	E $\flat$	A $\flat$	G	D $\flat$	D	F	F $\sharp$	R <sub>6</sub>
P <sub>4</sub>	D	A $\flat$	B $\flat$	G	A	D $\flat$	G $\flat$	F	B	C	E $\flat$	E	R <sub>4</sub>
P <sub>7</sub>	F	B	D $\flat$	B $\flat$	C	E	A	A $\flat$	D	E $\flat$	G $\flat$	G	R <sub>7</sub>
P <sub>5</sub>	E $\flat$	A	B	A $\flat$	B $\flat$	D	G	G $\flat$	C	D $\flat$	E	F	R <sub>5</sub>
P <sub>1</sub>	B	F	G	E	G $\flat$	B $\flat$	E $\flat$	D	A $\flat$	A	C	D $\flat$	R <sub>1</sub>
P <sub>8</sub>	F $\sharp$	C	D	B	D $\flat$	F	B $\flat$	A	E $\flat$	E	G	A $\flat$	R <sub>8</sub>
P <sub>9</sub>	G	D $\flat$	E $\flat$	C	D	G $\flat$	B	B $\flat$	E	F	A $\flat$	A	R <sub>9</sub>
P <sub>3</sub>	C $\sharp$	G	A	F $\sharp$	A $\flat$	C	F	E	B $\flat$	B	D	D $\sharp$	R <sub>3</sub>
P <sub>2</sub>	C	F $\sharp$	G $\sharp$	F	G	B	E	E $\flat$	A	B $\flat$	C $\sharp$	D	R <sub>2</sub>
P <sub>11</sub>	A	D $\sharp$	F	D	E	G $\sharp$	C $\sharp$	C	F $\sharp$	G	B $\flat$	B	R <sub>11</sub>
P <sub>10</sub>	G $\sharp$	D	E	C $\sharp$	D $\sharp$	G	C	B	F	F $\sharp$	A	B $\flat$	R <sub>10</sub>
	R <sub>I0</sub>	R <sub>I6</sub>	R <sub>I8</sub>	R <sub>I5</sub>	R <sub>I7</sub>	R <sub>I11</sub>	R <sub>I4</sub>	R <sub>I3</sub>	R <sub>I9</sub>	R <sub>I10</sub>	R <sub>I1</sub>	R <sub>I2</sub>	

## b. Using pitch class nomenclature

T	4	6	3	5	9	2	1	7	8	E	0
4	T	0	9	E	3	8	7	1	2	5	6
2	8	T	7	9	1	6	5	E	0	3	4
5	E	1	T	0	4	9	8	2	3	6	7
3	9	E	8	T	2	7	6	0	1	4	5
E	5	7	4	6	T	3	2	8	9	0	1
6	0	2	E	1	5	T	9	3	4	7	8
7	1	3	0	2	6	E	T	4	5	8	9
1	7	9	6	8	0	5	4	T	E	2	3
0	6	8	5	7	E	4	3	9	T	1	2
9	3	5	2	4	8	1	0	6	7	T	E
8	2	4	1	3	7	0	E	5	6	9	T

(T=10, E=11)

# Terminology

- **pitch class** — a particular pitch, identified by a name or number (e.g., D = pc 2) regardless of registral placement (octave equivalence).
- **interval class (ic)** — the distance between two pitches expressed numerically, without regard for spelling, octave compounding, or inversion (e.g., interval class 3 = minor third or major sixth).
- **pitch-class set** — collection of pitches expressed numerically, without regard for order or pitch duplication; e.g., “dominant 7th” chord = [0,4,7,10].
- **operations** — permutations of the pc set:
  - **transposition:** [0,4,7,10] — [1,5,8,11] — [2,6,9,0] — etc.
  - **retrograde:** [0,4,7,10] — [10,7,4,0]
  - **Inversion:** [0,4,7,10] inverts to [0,8,5,2] — *name that chord...?*
  - **rotation:** [0,4,7,10] — [4,7,10,0] — [7,10,0,4] — [10,0,4,7]
- **normal order** — the arrangement of a pc set in *ascending order* and in the most *intervallically compact* form: rotate [0,4,7,10] to [4,7,10,0]; then invert to [4,6,9,0] to determine **best normal order** (with the smallest intervals to the left).
- **prime form** — the transposition of the best normal form of the pc set beginning on 0: transpose [4,6,9,0] to [0,2,5,8] to get the prime form.



# Terminology

- **invariance** — sets that contain duplications under the operations of transposition and/or inversion (i.e., symmetrical pc sets).
- **subsets and supersets** — smaller pc sets contained within a larger set (subset) or larger sets that include smaller sets (superset): e.g., [0,2,5] of a **subset** of [0,2,5,8], while [0,1,2,5,7,8] is a **superset** that contains [0,2,5,8].
- **complementary set** — the set of pitch classes not present within any given pc set: e.g., the complement of [0,2,5,8] is [1,3,4,6,7,9,10,11] (prime form = [0,1,2,4,5,7,8,10]).
- **interval vector** — the array of possible intervals available within each pc set.
- **Z relation** — two pc sets that have the same interval vector.
- **K-relation and Kh-relation sets** — when a pc set occurs as a subset/superset of another pc set and/or its complement.
- **Forte set-class names** — Allen Forte in *The Structure of Atonal Music* (1973) organized all 3- to 9-note sets (expressed in prime form) and labeled them by cardinality (trichord, tetrachords, etc.); e.g., pc set [0,2,5,8] is labeled as pc set 4-27. Each pc set listing also includes the **interval vector** and **complementary set**, as well as any **inversional invariance** (indicated parenthetically) or **Z relation**.

# Prime Forms and Vectors of Pitch Class Sets (Allen Forte)

Name	Pcs	Vector	Name	Pcs	Vector
3-1(12)	0,1,2	210000	9-1	0,1,2,3,4,5,6,7,8	876663
3-2	0,1,3	111000	9-2	0,1,2,3,4,5,6,7,9	777663
3-3	0,1,4	101100	9-3	0,1,2,3,4,5,6,8,9	767763
3-4	0,1,5	100110	9-4	0,1,2,3,4,5,7,8,9	766773
3-5	0,1,6	100011	9-5	0,1,2,3,4,6,7,8,9	766674
3-6(12)	0,2,4	020100	9-6	0,1,2,3,4,5,6,8,10	686763
3-7	0,2,5	011010	9-7	0,1,2,3,4,5,7,8,10	677673
3-8	0,2,6	010101	9-8	0,1,2,3,4,6,7,8,10	676764
3-9(12)	0,2,7	010020	9-9	0,1,2,3,5,6,7,8,10	676683
3-10(12)	0,3,6	002001	9-10	0,1,2,3,4,6,7,9,10	668664
3-11	0,3,7	001110	9-11	0,1,2,3,5,6,7,9,10	667773
3-12(4)	0,4,8	000300	9-12	0,1,2,4,5,6,8,9,10	666963
4-1(12)	0,1,2,3	321000	8-1	0,1,2,3,4,5,6,7	765442
4-2	0,1,2,4	221100	8-2	0,1,2,3,4,5,6,8	665542
4-3(12)	0,1,3,4	212100	8-3	0,1,2,3,4,5,6,9	656542
4-4	0,1,2,5	211110	8-4	0,1,2,3,4,5,7,8	655552
4-5	0,1,2,6	210111	8-5	0,1,2,3,4,6,7,8	654553
4-6(12)	0,1,2,7	210021	8-6	0,1,2,3,5,6,7,8	654463
4-7(12)	0,1,4,5	201210	8-7	0,1,2,3,4,5,8,9	645652
4-8(12)	0,1,5,6	200121	8-8	0,1,2,3,4,7,8,9	644563
4-9(6)	0,1,6,7	200022	8-9	0,1,2,3,6,7,8,9	644464
4-10(12)	0,2,3,5	122010	8-10	0,2,3,4,5,6,7,9	566452
4-11	0,1,3,5	121110	8-11	0,1,2,3,4,5,7,9	565552
4-12	0,2,3,6	112101	8-12	0,1,3,4,5,6,7,9	556543
4-13	0,1,3,6	112011	8-13	0,1,2,3,4,6,7,9	556453
4-14	0,2,3,7	111120	8-14	0,1,2,4,5,6,7,9	555562
4-Z15	0,1,4,6	111111	8-Z15	0,1,2,3,4,6,8,9	555553
4-16	0,1,5,7	110121	8-16	0,1,2,3,5,7,8,9	554563
4-17(12)	0,3,4,7	102210	8-17	0,1,3,4,5,6,8,9	546652
4-18	0,1,4,7	102111	8-18	0,1,2,3,5,6,8,9	546553
4-19	0,1,4,8	101310	8-19	0,1,2,4,5,6,8,9	545752
4-20(12)	0,1,5,8	101220	8-20	0,1,2,4,5,7,8,9	545662
4-21(12)	0,2,4,6	030201	8-21	0,1,2,3,4,6,8,10	474643
4-22	0,2,4,7	021120	8-22	0,1,2,3,5,6,8,10	465562
4-23(12)	0,2,5,7	021030	8-23	0,1,2,3,5,7,8,10	465472
4-24(12)	0,2,4,8	020301	8-24	0,1,2,4,5,6,8,10	464743
4-25(6)	0,2,6,8	020202	8-25	0,1,2,4,6,7,8,10	464644
4-26(12)	0,3,5,8	012120	8-26	0,1,2,4,5,7,9,10	456562
4-27	0,2,5,8	012111	8-27	0,1,2,4,5,7,8,10	456553
4-28(3)	0,3,6,9	004002	8-28	0,1,3,4,6,7,9,10	448444
4-Z29	0,1,3,7	111111	8-Z29	0,1,2,3,5,6,7,9	555553

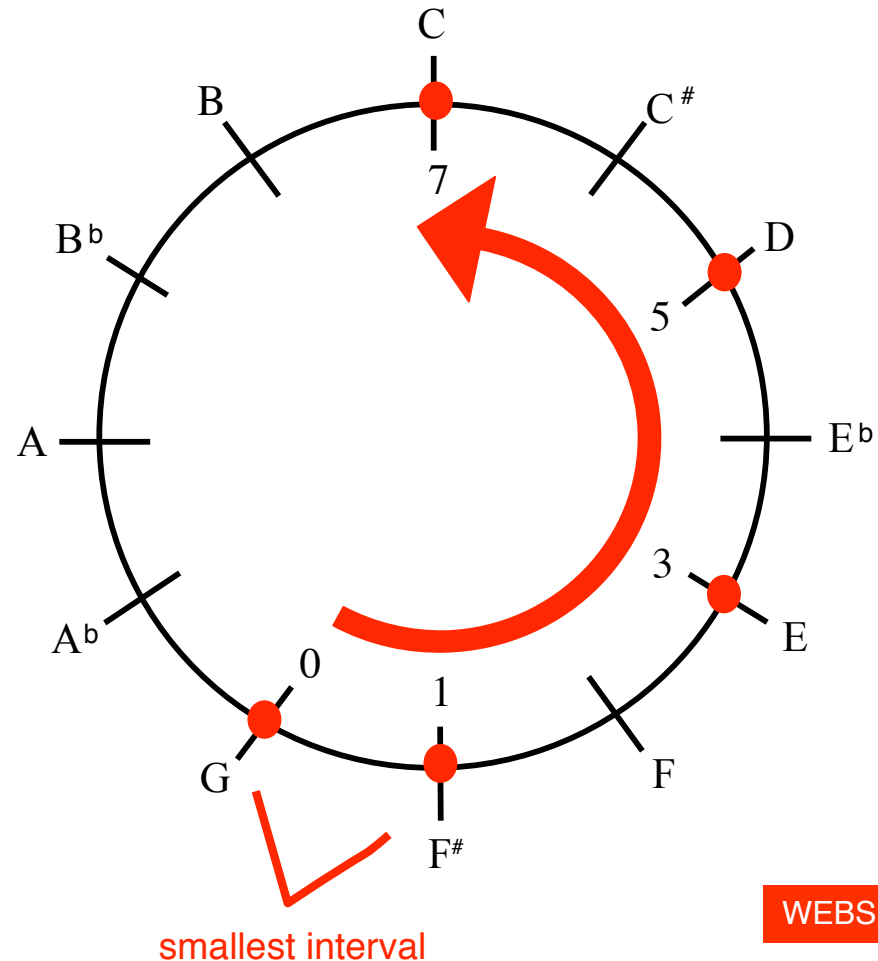
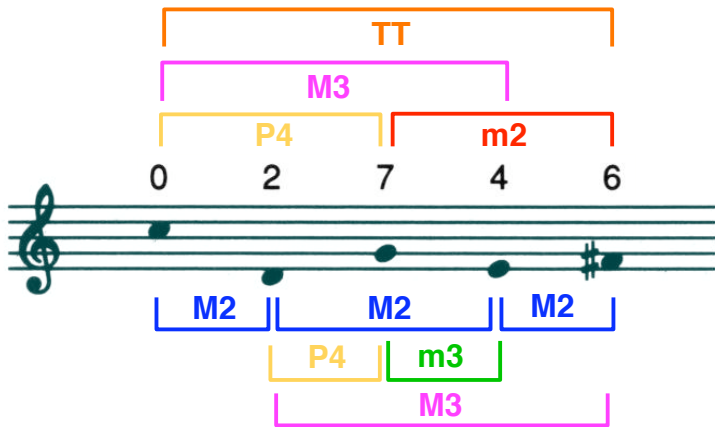
# Prime Forms and Vectors of Pitch Class Sets (Allen Forte)

Name	Pcs	Vector	Name	Pcs	Vector
5-1(12)	0,1,2,3,4	432100	7-1	0,1,2,3,4,5,6	654321
5-2	0,1,2,3,5	332110	7-2	0,1,2,3,4,5,7	554331
5-3	0,1,2,4,5	322210	7-3	0,1,2,3,4,5,8	544431
5-4	0,1,2,3,6	322111	7-4	0,1,2,3,4,6,7	544332
5-5	0,1,2,3,7	321121	7-5	0,1,2,3,5,6,7	543342
5-6	0,1,2,5,6	311221	7-6	0,1,2,3,4,7,8	533442
5-7	0,1,2,6,7	310132	7-7	0,1,2,3,6,7,8	532353
5-8(12)	0,2,3,4,6	232201	7-8	0,2,3,4,5,6,8	454422
5-9	0,1,2,4,6	231211	7-9	0,1,2,3,4,6,8	453432
5-10	0,1,3,4,6	223111	7-10	0,1,2,3,4,6,9	445332
5-11	0,2,3,4,7	222220	7-11	0,1,3,4,5,6,8	444441
5-Z12(12)	0,1,3,5,6	222121	7-Z12	0,1,2,3,4,7,9	444342
5-13	0,1,2,4,8	221311	7-13	0,1,2,4,5,6,8	443532
5-14	0,1,2,5,7	221131	7-14	0,1,2,3,5,7,8	443352
5-15(12)	0,1,2,6,8	220222	7-15	0,1,2,4,6,7,8	442443
5-16	0,1,3,4,7	213211	7-16	0,1,2,3,5,6,9	435432
5-Z17(12)	0,1,3,4,8	212320	7-Z17	0,1,2,4,5,6,9	434541
5-Z18	0,1,4,5,7	212221	7-Z18	0,1,2,3,5,8,9	434442
5-19	0,1,3,6,7	212122	7-19	0,1,2,3,6,7,9	434343
5-20	0,1,3,7,8	211231	7-20	0,1,2,4,7,8,9	433452
5-21	0,1,4,5,8	202420	7-21	0,1,2,4,5,8,9	424641
5-22(12)	0,1,4,7,8	202321	7-22	0,1,2,5,6,8,9	424542
5-23	0,2,3,5,7	132130	7-23	0,2,3,4,5,7,9	354351
5-24	0,1,3,5,7	131221	7-24	0,1,2,3,5,7,9	353442
5-25	0,2,3,5,8	123121	7-25	0,2,3,4,6,7,9	345342
5-26	0,2,4,5,8	122311	7-26	0,1,3,4,5,7,9	344532
5-27	0,1,3,5,8	122230	7-27	0,1,2,4,5,7,9	344451
5-28	0,2,3,6,8	122212	7-28	0,1,3,5,6,7,9	344433
5-29	0,1,3,6,8	122131	7-29	0,1,2,4,6,7,9	344352
5-30	0,1,4,6,8	121321	7-30	0,1,2,4,6,8,9	343542
5-31	0,1,3,6,9	114112	7-31	0,1,3,4,6,7,9	336333
5-32	0,1,4,6,9	113221	7-32	0,1,3,4,6,8,9	335442
5-33(12)	0,2,4,6,8	040402	7-33	0,1,2,4,6,8,10	262623
5-34(12)	0,2,4,6,9	032221	7-34	0,1,3,4,6,8,10	254442
5-35(12)	0,2,4,7,9	032140	7-35	0,1,3,5,6,8,10	254361
5-Z36	0,1,2,4,7	222121	7-Z36	0,1,2,3,5,6,8	444342
5-Z37(12)	0,3,4,5,8	212320	7-Z37	0,1,3,4,5,7,8	434541
5-Z38	0,1,2,5,8	212221	7-Z38	0,1,2,4,5,7,8	434442

# Prime Forms and Vectors of Pitch Class Sets (Allen Forte)

Name	Pcs	Vector	Name	Pcs
6-1(12)	0,1,2,3,4,5	543210		
6-2	0,1,2,3,4,6	443211		
6-Z3	0,1,2,3,5,6	433221	6-Z36	0,1,2,3,4,7
6-Z4(12)	0,1,2,4,5,6	432321	6-Z37(12)	0,1,2,3,4,8
6-5	0,1,2,3,6,7	422232		
6-Z6(12)	0,1,2,5,6,7	421242	6-Z38(12)	0,1,2,3,7,8
6-7(6)	0,1,2,6,7,8	420243		
6-8(12)	0,2,3,4,5,7	343230		
6-9	0,1,2,3,5,7	342231		
6-Z10	0,1,3,4,5,7	333321	6-Z39	0,2,3,4,5,8
6-Z11	0,1,2,4,5,7	333231	6-Z40	0,1,2,3,5,8
6-Z12	0,1,2,4,6,7	332232	6-Z41	0,1,2,3,6,8
6-Z13(12)	0,1,3,4,6,7	324222	6-Z42(12)	0,1,2,3,6,9
6-14	0,1,3,4,5,8	323430		
6-15	0,1,2,4,5,8	323421		
6-16	0,1,4,5,6,8	322431		
6-Z17	0,1,2,4,7,8	322332	6-Z43	0,1,2,5,6,8
6-18	0,1,2,5,7,8	322242		
6-Z19	0,1,3,4,7,8	313431	6-Z44	0,1,2,5,6,9
6-20(4)	0,1,4,5,8,9	303630		
6-21	0,2,3,4,6,8	242412		
6-22	0,1,2,4,6,8	241422		
6-Z23(12)	0,2,3,5,6,8	234222	6-Z45(12)	0,2,3,4,6,9
6-Z24	0,1,3,4,6,8	233331	6-Z46	0,1,2,4,6,9
6-Z25	0,1,3,5,6,8	233241	6-Z47	0,1,2,4,7,9
6-Z26(12)	0,1,3,5,7,8	232341	6-Z48(12)	0,1,2,5,7,9
6-27	0,1,3,4,6,9	225222		
6-Z28(12)	0,1,3,5,6,9	224322	6-Z49(12)	0,1,3,4,7,9
6-Z29(12)	0,1,3,6,8,9	224232	6-Z50(12)	0,1,4,6,7,9
6-30(12)	0,1,3,6,7,9	224223		
6-31	0,1,3,5,8,9	223431		
6-32(12)	0,2,4,5,7,9	143250		
6-33	0,2,3,5,7,9	143241		
6-34	0,1,3,5,7,9	142422		
6-35(2)	0,2,4,6,8,10	060603		

# Deriving Pitch-class Sets—Clock Method



WEBSITE

- Plot the pitches on a clock graph
- Determine the *largest* space between adjacent points; of these two points, find the *smallest* adjacent interval in the opposite direction.
- Pitches will be labeled in the direction of this small interval; begin labeling with zero [0] and count upwards from there.
- Identify pitch class set on Forte chart.

$[0,1,3,5,7] = \text{pc set 5-24}$

interval vector =  $\begin{matrix} & \text{m2} & \text{M2} & \text{m3} & \text{M3} & \text{P4} & \text{TT} \\ = & 1 & 3 & 1 & 2 & 2 & 1 \end{matrix}$





Igor Stravinsky (left) and Elliott Carter in 1961.

# Application of Pitch Class Sets in Igor Stravinsky's *Three Songs from William Shakespeare, No. 1* (1953)

Pitch-class set **4-2** [0, 1, 2, 4]

The musical score is annotated with interval classes. Above the staff, blue brackets labeled 'R' and orange brackets labeled 'RI' indicate intervals of 1, 2, and 4. Below the staff, red brackets labeled 'P' and purple brackets labeled 'I' indicate intervals of 4, 2, and 1. A green bracket at the bottom groups the first six notes as 'P' and the last six as 'I'. A vertical dashed green line separates the two segments.

- Chain of interval classes [1], [2], and [4]
- Inversional symmetry between segments

# Pitch Class Set Analysis of Alban Berg's *Wozzeck* (Act II)

4-19 : [8,9,0,4]      4-4 : [10,1,2,3]      5-10 : [11,1,2,4,5]      6-Z44 : [1,2,3,6,7,10]

7-4 : (6,7,8,9,0)      4-Z15 : [11,1,4,5]      6-Z19 : [2,3,6,7,9,10]      7-5 : [6,7,8,10,11,0,1]

4-Z15 : [2,3,6,8]      5-16 : [3,6,7,9,10]

367 *Largo* (♩ = 44 - 48)

Cl. Bn.      Ob. E.H. Hn.

Vc.

5-21 : [8,9,0,1,4]      5-30 : [8,9,0,2,4]      7-15 : [8,9,10,0,2,3,4]      5-9 : [2,3,4,6,8]      4-Z29 : [3,7,9,10]      5-21 : [2,3,6,7,10]      4-16 : [6,8,0,1]

5-10 : [4,6,7,9,10]

5-5 : [4,8,9,10,11]      4-Z29 : [4,8,10,11]

5-27 : [0,3,5,7,8]

370

Hn.      Cl.      Cb.

5-19 : [3,4,7,9,10]      4-16 : [7,8,0,2]      4-14 : [5,7,8,0]

7-9 : (9,10,11,1,3)      6-22 : [0,2,4,6,7,8]      5-30 : [0,2,4,7,8]      5-30 : [0,2,4,7,8]

(8,11,0,1,2)