KEK Internal 2000-20
March 2001
R

Lecture Notes of Radiation Transport Calculation by Monte Carlo Method

(English Version)
Revised 10/11/2001

H. Hirayama and Y. Namito

High Energy Accelerator Reserach Organization
1-1 Oho, Tsukuba-shi, Ibaraki, 305-0801 Japan

Contents

English Parts 1

1 Monte Carlo Method 1

1.1 Random numbers . . . 1
1.2 Pseudo random numbers . . . 1
1.3 Production of pseudo random numbers using a pocket calculator . . . 2
1.4 Calculation of π using random numbers . . . 2

2 Radiation Transport by the Monte Carlo Method 4

2.1 Sampling method . . . 4
2.1.1 Continuous probability process . . . 4
2.1.2 Discrete probability process . . . 5
2.2 Simulation of radiation transport inside media . . . 7

3 A Simple Example of Radiation Transport 8

3.1 Single layer . . . 8
3.2 Double layer . . . 8

4 Complex, but More Realistic, Example of Radiation Transport 10

4.1 Example . . . 10
4.2 Practices . . . 12

English Parts

1 Monte Carlo Method

A method used to solve a problem with random numbers is called a "Monte Carlo Method".

1.1 Random numbers

Random numbers are a key tool for the Monte Carlo method. It is required to produce random numbers quickly when necessary. There are several ways to produce random numbers:

1. Use a dice, a roulette etc. — very slow.
2. Use a table of random numbers.
   - A table of random numbers has been well examined concerning its statistical characteristics.
   - It is required to store a whole table in computer data storage.
   - It currently is not very fast to produce random numbers.
3. Use physical random numbers like the decay of a radioisotope.
   - It is not easy to digitalize, and has a weakness concerning stability and reproducibility.
4. Produce random numbers successively from a seed random number, R_0, using a recurrence formula (a congruence equation in ordinary) in the form of $R_{n+1} = f(R_n)$. (pseudo-random numbers).
   - It is possible to produce the same random number sequences if the seed random number is the same.
   - Pseudo random numbers residuals by a divider, m.
   - There are m different integers at most and, therefore, pseudo random numbers have a limited period.
   - Good pseudo random numbers have the following features:
     (a) fast to create a random number
     (b) a long sequence
     (c) reproducibility
     (d) good statistical characteristics
   - It is possible to create pseudo random numbers between 0 and 1 by dividing pseudo random numbers by m.
5. There is another type of random-number generator called the Marasaglia-Zaman random-number generator[1]. It has a long periodicity ($2^{144} \sim 10^{43}$), and is portable to all 32-bit machines.

1.2 Pseudo random numbers

A linear congruence methods proposed by D. H. Lehmer is most widely used to produce pseudo random numbers:

$$R_{n+1} \equiv mod(aR_n + b, m) \qquad (n = 0, 1, ..., m),$$

where a, b and m are positive integers and a divider m is the length of the integer value allowed in the compiler ($m = 2^{31}$ is used for a 32 bit case).

Pseudo random numbers frequently used in Monte Carlo calculations and their a, b and m are given in Table 1.

Table 1. Names of pseudo random numbers and their a, b and m.

Name	a	b	m
RANDU	65539	0	2^{31}
SLAC RAN1	69069	0	2^{31}
SLAC RAN6	663608491	0	2^{31}

1.3 Production of pseudo random numbers using a pocket calculator

1. Produce 10 random numbers for $R_0 = 3$, $a = 5$ and $m = 16$.
2. Confirm that the same sequence appears from some point.
   A number of random numbers produced until the same sequence appears is called a "sequence".
3. What is a sequence in this case ?
4. Check for a different R_0.

n	R_n	$R_n * 5$	$R_{n+1} = mod(R_0 * 5, 16)^*$	R_n	$R_n * 5$	$R_{n+1} = mod(R_0 * 5, 16)$
0	3					
1						
2						
3						
4						
5						
6						
7						
8						
9						
10						

$^*mod(R_0 * 5, 16) = R_0 * 5 - INT(\frac{R_0*5}{16}) * 16$

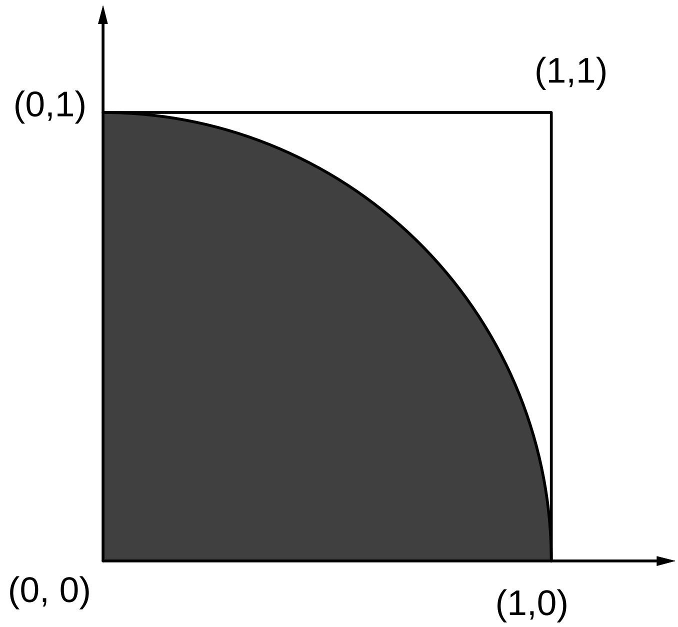
1.4 Calculation of π using random numbers

Select 2 random numbers between 0 and 1 in order starting from an arbitrary place in Table 2, which is created by SLAC RAN6, and count the number of pairs which satisfy the following condition.

$$R = \sqrt{\xi^2 + \eta^2} \leq 1.0$$

Trial number	ξ	η	R	$R \leq 1$
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				
				(A)
A/10=		(A/10)*4=		

A fraction ($A/10$) which satisfies the condition corresponds to the area within a circle of radius 1cm in a square of 1 cm. This is $\pi/4$ and, therefore, $\pi = 4 \times A/10$.



2 Radiation Transport by the Monte Carlo Method

Radiation trajectories are followed in a Mote Carlo calculation by determining each physical process with probability variables which describe each process.

2.1 Sampling method

2.1.1 Continuous probability process

A probability distribution function (PDF:$f(x)$) for each physical process is defined over the range $[a, b]$, where neither a nor b is necessary finite. A PDF must have the properties such that it is both integrable and non-negative.

We now construct its cumulative probability function (CDF:$F(x)$),

$$F(x) = \int_a^x f(x_i)dx_i,$$

and assume that it is properly normalized, *i.e.* $F(b) = 1$.

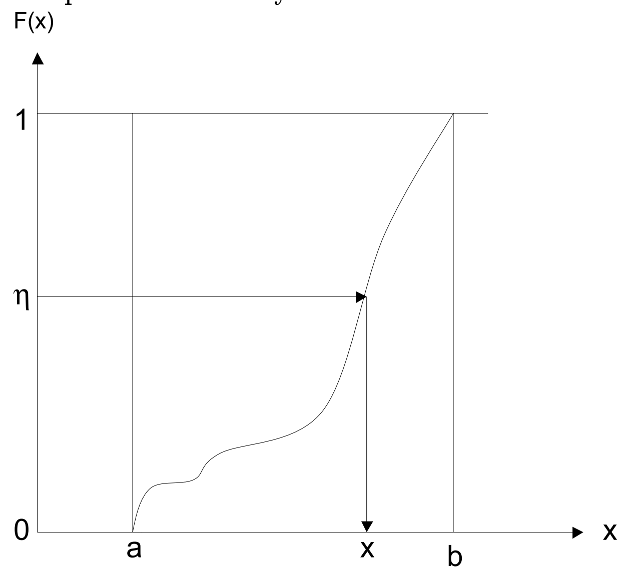
By its definition, we can map $F(x)$ onto a range of random variables, η, where $0 \leq \eta \leq 1$.

Having mapped the random numbers onto $F(x)$, we may invert the equation to give

$$x = F^{-1}(\eta).$$

The way to determine x by solving the above equation is called a "direct method".

In general, various techniques are necessary to determine x from the above equation.



Example of a direct method–determination of flight distance A particle interaction position is determined as follows:

1. If the interaction probability of a particle per unit distance is Σ_t, the number of decreases (dn) after dl is given by

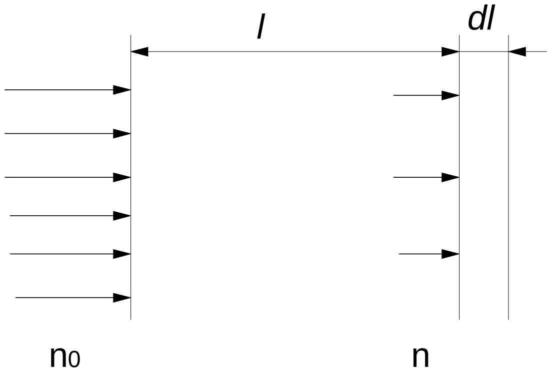
$$dn = -n\Sigma_t dl.$$

Therefore,

$$\int_{n_0}^{n} \frac{dn}{n} (= \ln \frac{n}{n_0}) = \int_0^l (-\Sigma_t)dl (= -\Sigma_t l),$$

$$\frac{n}{n_0} = e^{-\Sigma_t l},$$

where n_0 is the number of particles at $l = 0$.



2. $e^{-\Sigma_t l}$ is the probability that a particle does not interact within distance l. Therefore, the probability that a first interaction occurs between l and $l + dl$ is

$$p(l)dl = e^{-\Sigma_t l}\Sigma_t dl$$

and

$$\eta = P(l) = \int_0^l p(l_1)dl_1 = 1 - e^{-\Sigma_t l},$$

where η is a random number between 0 and 1.[1]

3. By solving this equation, the flight distance (l) can be determined as

$$l = -\frac{1}{\Sigma_t}\ln(1-\eta) = -\lambda\ln(1-\eta).$$

$\lambda = 1/\Sigma_t$ is called as the "mean free path".

4. Considering that $1-\eta$ is equivalent to η, l is usually determined by

$$l = -\lambda\ln\eta.$$

2.1.2 Discrete probability process

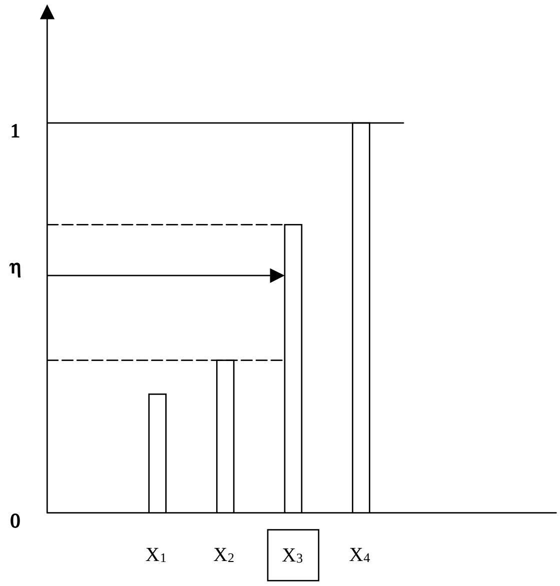
If a probability variable (x) takes on discrete values (x_i) with probabilities (p_i) such that

$$F(x_n) = \sum_{i=1}^{n} p_i = 1,$$

$x = x_i$ if

$$F(x_i) = \sum_{j=1}^{i} p_j \le \eta < F(x_{i+1}) = \sum_{j=1}^{i+1} p_j,$$

[1] $\int_0^\infty p(l)dl = 1$



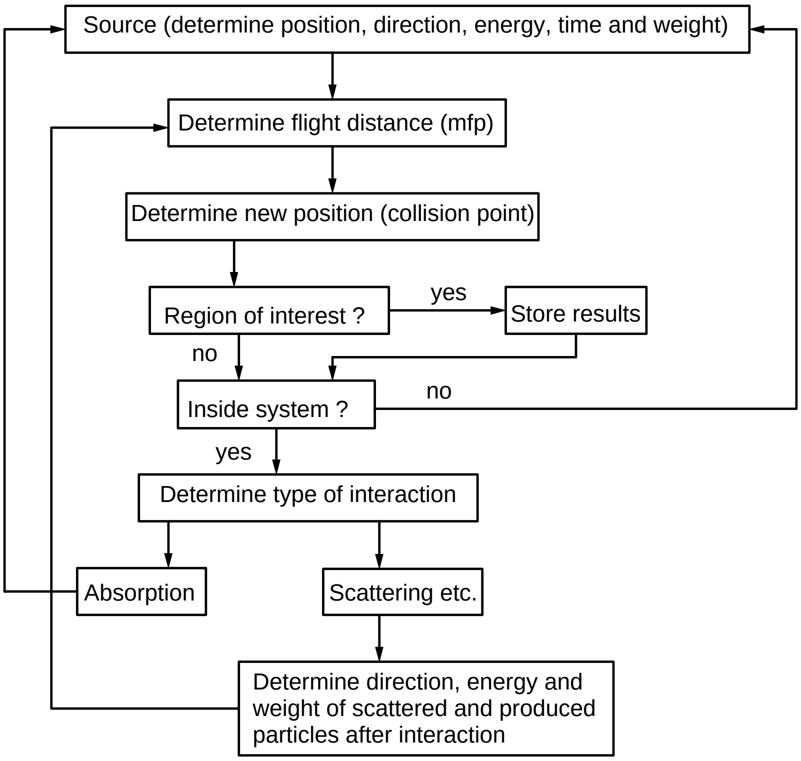
Example of a discrete probability process The probabilities of the photoelectric effect, Compton scattering and pair creation at a photon interaction are P_{photo}, P_{Compt} and P_{pair}, respectively.

- $P_{photo} + P_{Compt} + P_{pair} = 1.0$
- If $\eta \leq P_{photo}$, the reaction is a photoelectric.
- If $P_{photo} < \eta \leq P_{photo} + P_{Compt}$, the reaction is Compton scattering.
- If $P_{photo} + P_{Compt} < \eta$, the reaction is pair creation.

2.2 Simulation of radiation transport inside media

Source radiation simultaneously moves inside media while changing its position, direction and energy by scattering until it is absorbed. It is possible to obtain information like the number of particles or the absorbed energy at a specified region by the Monte Carlo method.

A basic flowchart of the Monte Carlo method is as follows:



1. Determine the source parameters.
   - position coordinates
   - direction coordinates
   - energy
   - weight
2. Determine the distance to a interaction point, the flight distance (l), using the total cross section.
3. Check whether an interaction point is within the same region or not.
   - Uncharged particle, like photons or neutrons, move to an interaction point without changing its direction or energy. In this case, this is a comparison between the flight distance (l) and the distance to the region boundary (d).
     (a) If $l < d$, move the particle to the interaction point.
     (b) If $l \geq d$, move the particle to the boundary.
       – If the medium of the new region is the same, set the flight distance to $l - d$ and repeat the same procedure. Otherwise, determine the flight path for the new medium.
       – If the new region is outside the system of interest, stop following this particle and produce a new particle.

- A charged particle, like an electron, changes its direction and energy while moving to the interaction point and, therefore, treatments become more complicate.

4. Determine the type of interaction.
   - The type of interaction is determined using discrete-type probability distribution functions.
   - Photoelectric or Compton scattering or pair production is selected in the case of photons.
5. Determine the energy and a direction of scattered and produced particles at the interaction point using the differential cross section of the interaction.
6. Store any information of interest when a particle reaches to region of interest, such as:
   - type of particle and its energy,
   - energy imparted to the medium.
7. Terminate following radiation when
   - radiation leaks from the system or
   - the radiation energy becomes below its cut-off energy.
8. A history is defined as the whole processes from the production of a source particle until its termination for some reason. Information of interests can be obtained by repeating a history many times.

3 A Simple Example of Radiation Transport

3.1 Single layer

Consider uniform medium, A, of 50 cm thickness (see Fig. 1).

1. Suppose that
   - 0.5MeV photons enter on this system from the left end,
   - the mean free path is 20 cm,
   - the ratio of the photoelectric effect and Compton scattering is 1:1, and
   - a scattered photon does not change its energy or direction.
2. Starting from an arbitrary random number in Table 2, follow 10 photons like an example in Table 3, and count the number of photons transmitted in a plane.
3. Make trajectories of particles like an example in Fig. 1.

3.2 Double layer

Consider 40 cm of medium A followed by 10 cm of medium B (see Fig. 2).

1. Suppose that:
   - 0.5MeV photons enter this system from the left end,
   - the mean free path and the ratio of the photoelectric effect and Compton scattering in medium A are same as in the previous case,

- the mean free path of medium B is 3cm,
- the ratio of the photoelectric effect and Compton scattering of medium B is 3:1, and
- a scattered photon does not change its energy or direction for both media.

2. Starting from an arbitrary random number in Table 2, follow 10 photons, like the example in Table 4, and count the number of photons transmitted in medium B.

3. Make trajectories of particles, like the example given in Fig. 2.

4 Complex, but More Realistic, Example of Radiation Transport

Consider the 10 cm aluminum plane shown in Fig. 3.

Suppose that

1. 0.5MeV photons enter this system from the left end,
2. Photons are scattered with equal probability for each 45° at Compton scattering for all photon energies,

Scattering Angle	Probability
0°	20%
45°	20%
90°	20%
135°	20%
180°	20%

3. The photon energy after scattering is calculated by

$$E = \frac{E_0}{1 + \left(\frac{E_0}{0.511}\right)(1 - \cos\theta)},$$

where E_0(MeV) is the photon energy before scattering, E(MeV) is that after scattering and θ is the scattering angle.

4. Suppose that the azimuthal angle after Compton scattering is 0° or 180° with an equal probability. 0° is 90° left from the particle direction and 180° is 90° right.
5. Use the mean free path (mfp) and branching ratio for each photon energy in Figs. 4 and 5.
6. Set the cutoff energy of photons to 0.05MeV.

4.1 Example

The example in Table 5 can be explained as follows:

- Source photon
  1. The mfp of 0.5MeV is 4.15cm from Fig. 4.
  2. If we start a random number from 0.35139 in Table 3, the flight distance of this photon is
  $$l = -\ln(0.35139) * 4.15 = 4.34(cm).$$
  3. This distance is smaller than that to the boundary (10cm). The reaction point is therefore inside the Al plane.
  4. The probability of a photoelectric reaction for 0.5MeV is 0.0018 from Fig. 5.
  5. The next random number is 0.25872, which is larger than 0.0018. Therefore, the reaction is Compton scattering.
  6. Next, determine the scattering angle. The scattering angle is 0° if a random number is smaller than 0.2, 45° if it is between 0.2 and 0.4, 90° if it is between 0.4 and 0.6, 135° if it is between 0.6 and 0.8 and 180° if it is larger than 0.8. The next random number is 0.57197. Therefore, the scattering angle is 90°.

7. Calculate photon energy after scattering.

$$E = \frac{0.5}{1 + \left(\frac{0.5}{0.511}\right)(1 - \cos 90^\circ)} = 0.25(MeV)$$

8. The azimuthal angle is 0° if the random number is less than 0.5, and is 180° otherwise. The next random number is 0.88784, and therefore the azimuthal number is 180°.

- Scattered photon after the first interaction

1. The mfp of 0.25MeV is 3.34cm from Fig. 4.
2. The next random number is 0.23809 and the flight distance is

$$l = -\ln(0.23809) * 3.34 = 4.79(cm).$$

3. The plane is infinite for the X-direction. Therefore, an interaction occurs within the Al plane.
4. The probability of a photoelectric reaction for 0.25MeV is 0.01 from Fig. 5.
5. The next random number is 0.66926, which is larger than 0.01. Therefore, the reaction is Compton scattering.
6. The next random number is 0.047825 and the scattering angle is 0°. For $\theta = 0^\circ$, a photon does not change energy and it is not necessary to determine the azimuthal angle.
7. The photon moves from the position of X=-4.79 cm and Z=4.34 cm to the direction of -X.

- Scattered photon after a second interaction

1. The mfp of 0.25MeV is 3.34cm, the same as in the previous case.
2. The next random number is 0.94933 and the flight distance is

$$l = -\ln(0.94933) * 3.34 = 0.17(cm).$$

3. The plane is infinite for the X-direction. Therefore, an interaction occurs within the Al plane.
4. The probability of a photoelectric reaction for 0.25MeV is 0.01, the same as in the previous case.
5. The next random number is 0.32386, which is larger than 0.01. Therefore, the reaction is Compton scattering.
6. The next random number is 0.57888 and the scattering angle is 90°.
7. Calculate the photon energy after scattering,.

$$E = \frac{0.25}{1 + \left(\frac{0.25}{0.511}\right)(1 - \cos 90^\circ)} = 0.17(MeV).$$

8. The next random number is 0.43852, which is smaller than 0.5. Therefore, the azimuthal angle is 0°.
9. The photon moves from the positions of X=-4.96 cm and Z=4.34 cm to the direction of Z.

4.2 Practices

1. Following the same procedure as shown above until a photoelectric effect occurs, the photon energy becomes below a cut-off energy or the photo reaches the boundary (Z $<$ 0.0 or Z $>$ 10 cm).

2. Start from another source photon and follow its movements as in the above example. Make trajectories of the photon in Fig. 6, like the example in Fig. 3.

3. Change the medium from Al to Fe. Start from a source photon and follow its movements, as in the above example. Make trajectories of the photon in Fig. 6 like the example in Fig. 3.

References

[1] G. Masaglia and A. Zaman, "A New Class of Random Number Generator", *Annals of Applied Probability* **1**(1991)462-480.

Table 2.a Pseudo random number between 0–1 (RAN6).

□□□0.35139	□□□0.80759E-01	□□□0.87901	□□□0.14683	□□□0.35139
□□□0.25872	□□□0.72516E-01	□□□0.58570	□□□0.94936E-01	□□□0.50826
□□□0.57197	□□□0.49574	□□□0.64058	□□□0.61157	□□□0.87163
□□□0.88784	□□□0.88258	□□□0.69760	□□□0.36039	□□□0.35879
□□□0.23809	□□□0.27791	□□□0.85864	□□□0.73451	□□□0.39308
□□□0.66926	□□□0.30750	□□□0.31889	□□□0.44235E-01	□□□0.99049
□□□0.47825E-01	□□□0.43553	□□□0.57450	□□□0.92100	□□□0.12666
□□□0.94993	□□□0.59972	□□□0.40857	□□□0.68448E-01	□□□0.45970
□□□0.32386	□□□0.51381	□□□0.54860	□□□0.84018	□□□0.64031
□□□0.57888	□□□0.40695	□□□0.83314	□□□0.91684	□□□0.90787
□□□0.43852	□□□0.36577	□□□0.40154	□□□0.14651	□□□0.39914
□□□0.28781	□□□0.18749	□□□0.54845	□□□0.66338	□□□0.21408
□□□0.67339	□□□0.31589	□□□0.97238	□□□0.15993	□□□0.45788
□□□0.20186	□□□0.70900	□□□0.65725	□□□0.60494	□□□0.56173
□□□0.13777	□□□0.38382E-01	□□□0.21781	□□□0.41416E-01	□□□0.47288E-01
□□□0.57554	□□□0.25912	□□□0.89899E-01	□□□0.23234	□□□0.16318
□□□0.55449	□□□0.48333	□□□0.25344	□□□0.23082	□□□0.91735
□□□0.48447E-01	□□□0.91123	□□□0.57389	□□□0.33224	□□□0.77268
□□□0.87311	□□□0.78998	□□□0.13426	□□□0.48848	□□□0.88988
□□□0.80901E-02	□□□0.12214	□□□0.31935	□□□0.39584E-01	□□□0.59837
□□□0.54234	□□□0.75120E-01	□□□0.43625	□□□0.54979	□□□0.88178
□□□0.78393	□□□0.84099	□□□0.20327	□□□0.40666	□□□0.54844
□□□0.61293	□□□0.27865E-01	□□□0.38489	□□□0.91372	□□□0.43747
□□□0.12729	□□□0.92955	□□□0.93441	□□□0.26941	□□□0.16451
□□□0.40105	□□□0.12858	□□□0.92839	□□□0.39179	□□□0.30527
□□□0.50580	□□□0.78422	□□□0.47366	□□□0.62539	□□□0.85695
□□□0.96950	□□□0.79039	□□□0.52472	□□□0.49319	□□□0.55714
□□□0.90944	□□□0.35825	□□□0.62174	□□□0.14015	□□□0.91235
□□□0.93012	□□□0.54581	□□□0.25136	□□□0.26461	□□□0.93489
□□□0.87628	□□□0.60426E-01	□□□0.91601	□□□0.51969	□□□0.94257
□□□0.78570	□□□0.31895	□□□0.90516	□□□0.59026	□□□0.25842
□□□0.52241E-01	□□□0.23444E-02	□□□0.55293	□□□0.76777E-01	□□□0.73780
□□□0.96322	□□□0.58566	□□□0.78733	□□□0.93790	□□□0.60238

Table 2.b Pseudo random number between 0–1 (RAN6).

□□□0.54787	□□□0.53120	□□□0.72037	□□□0.62719	□□□0.62270
□□□0.59271	□□□0.57923	□□□0.61416	□□□0.86082	□□□0.28776
□□□0.39830	□□□0.66884	□□□0.23577	□□□0.62964	□□□0.14532
□□□0.70681	□□□0.39862	□□□0.20961	□□□0.98400	□□□0.29331
□□□0.31935	□□□0.72989	□□□0.49154	□□□0.97889	□□□0.24027E-02
□□□0.62644	□□□0.66915	□□□0.68759E-01	□□□0.46399	□□□0.21913
□□□0.43298	□□□0.76310	□□□0.15859	□□□0.82671	□□□0.27310
□□□0.60609	□□□0.42198E-01	□□□0.28413	□□□0.76788	□□□0.18095
□□□0.45978	□□□0.29978	□□□0.64135	□□□0.98860	□□□0.44425E-01
□□□0.89927E-01	□□□0.51068E-01	□□□0.44746	□□□0.81574	□□□0.72815
□□□0.79392	□□□0.95171	□□□0.66497	□□□0.64700	□□□0.11741
□□□0.92583	□□□0.41429	□□□0.72960	□□□0.82387	□□□0.81259
□□□0.63791	□□□0.42849	□□□0.35523	□□□0.93218	□□□0.43228
□□□0.34941	□□□0.54567	□□□0.14993	□□□0.32309	□□□0.73736
□□□0.72977	□□□0.29015	□□□0.49649	□□□0.30255	□□□0.89366
□□□0.23354	□□□0.54200	□□□0.82362	□□□0.55659	□□□0.63567
□□□0.89462	□□□0.96583	□□□0.70504E-01	□□□0.20410	□□□0.16373
□□□0.75146	□□□0.25056E-01	□□□0.47159	□□□0.53616	□□□0.12013
□□□0.44562	□□□0.28374E-02	□□□0.44094	□□□0.16473E-01	□□□0.47173
□□□0.97241	□□□0.66338	□□□0.44258	□□□0.20358	□□□0.51183E-01
□□□0.95758E-01	□□□0.91285	□□□0.40385	□□□0.53894	□□□0.31227
□□□0.74870	□□□0.74263	□□□0.68049	□□□0.15573	□□□0.65054
□□□0.27272	□□□0.10299	□□□0.52343	□□□0.98467	□□□0.82302
□□□0.31172	□□□0.53977	□□□0.22246	□□□0.99720	□□□0.18207
□□□0.30305	□□□0.96944	□□□0.46553	□□□0.38509	□□□0.39407
□□□0.21660E-01	□□□0.23708	□□□0.68408	□□□0.33383	□□□0.88696
□□□0.59989	□□□0.39838E-01	□□□0.17807	□□□0.20854	□□□0.41660
□□□0.46197	□□□0.43592	□□□0.52838	□□□0.46316	□□□0.54383
□□□0.50037	□□□0.82801	□□□0.49781	□□□0.61846	□□□0.77787
□□□0.28417	□□□0.74824	□□□0.47328	□□□0.70469	□□□0.67670
□□□0.50405	□□□0.56071	□□□0.83753	□□□0.88639	□□□0.50228
□□□0.64247	□□□0.20578	□□□0.92012	□□□0.79337	□□□0.80499
□□□0.11305	□□□0.90084	□□□0.77510	□□□0.78337	□□□0.57539
□□□0.45989	□□□0.81984E-01	□□□0.53143	□□□0.58375	□□□0.96921
□□□0.17654E-01	□□□0.97539	□□□0.37816	□□□0.50861E-01	□□□0.21769
□□□0.88863	□□□0.92111	□□□0.80135	□□□0.23045	□□□0.82503
□□□0.75763	□□□0.16838	□□□0.70333	□□□0.48403E-01	□□□0.44966
□□□0.91739	□□□0.86200	□□□0.39556	□□□0.77209	□□□0.62544
□□□0.97018	□□□0.10432E-01	□□□0.85798	□□□0.13995	□□□0.45725
□□□0.88676	□□□0.48060	□□□0.93983	□□□0.40146	□□□0.15697
□□□0.65957	□□□0.83634	□□□0.56018E-01	□□□0.64547E-01	□□□0.77886
□□□0.45141	□□□0.10571	□□□0.55754	□□□0.40384	□□□0.91072

Table 2.c Pseudo random number between 0–1 (RAN6).

□□□0.34143	□□□0.44069	□□□0.98520	□□□0.18921	□□□0.44024
□□□0.23586E-01	□□□0.63700	□□□0.54632	□□□0.53836	□□□0.20249
□□□0.17648	□□□0.48868	□□□0.28461	□□□0.91320	□□□0.61306
□□□0.69758	□□□0.61872E-01	□□□0.89250	□□□0.27406	□□□0.35883
□□□0.63995E-01	□□□0.68577	□□□0.75469	□□□0.33241	□□□0.91565
□□□0.83815	□□□0.87970	□□□0.59948	□□□0.52269	□□□0.20673
□□□0.86424	□□□0.42430	□□□0.84045	□□□0.33149	□□□0.86152
□□□0.66837	□□□0.24751	□□□0.80217	□□□0.84606E-01	□□□0.69456
□□□0.21250	□□□0.55885	□□□0.68996	□□□0.29841	□□□0.94124
□□□0.63574E-01	□□□0.61021	□□□0.10448	□□□0.69198	□□□0.28055
□□□0.52365	□□□0.86484	□□□0.44606	□□□0.40250	□□□0.14792
□□□0.97542E-01	□□□0.62146	□□□0.26055	□□□0.45429E-01	□□□0.50240
□□□0.51699	□□□0.35525	□□□0.12800	□□□0.59158	□□□0.17429
□□□0.44616	□□□0.48223E-01	□□□0.97258	□□□0.34535	□□□0.63757
□□□0.66503	□□□0.72099	□□□0.25307	□□□0.89776	□□□0.77101
□□□0.28072	□□□0.83216	□□□0.94936	□□□0.26887	□□□0.32891
□□□0.13812	□□□0.46840	□□□0.98474	□□□0.43290E-01	□□□0.61160
□□□0.52931	□□□0.30556	□□□0.20938	□□□0.88584	□□□0.23360
□□□0.82830	□□□0.60500	□□□0.82444	□□□0.45963	□□□0.20830
□□□0.99595E-01	□□□0.47722	□□□0.67094	□□□0.39442	□□□0.90602
□□□0.64148	□□□0.72072	□□□0.43834	□□□0.42202	□□□0.71624
□□□0.58980	□□□0.65631E-01	□□□0.99181E-01	□□□0.53697	□□□0.99585
□□□0.19598	□□□0.77663	□□□0.87830	□□□0.81104	□□□0.60020
□□□0.91714	□□□0.31211	□□□0.22589	□□□0.88143	□□□0.18307
□□□0.10870	□□□0.59667	□□□0.96805	□□□0.78959	□□□0.86838

Table 3 Single layer.

No.	d(cm)	Random No.	l (cm)	$d > l$	$d \leq l$	Random No.	Photo	Compton
Exp. 1	50.0	0.2336	29.08	*		0.20830	*	
Exp. 2	50.0	0.90602	1.97	*		0.71624		*
	48.03	0.99585	0.083	*		0.60020		*
	47.95	0.18307	33.96	*		0.86838		*
	13.99	0.35139	20.92		*			

Fig. 1 Trajectories for a single layer

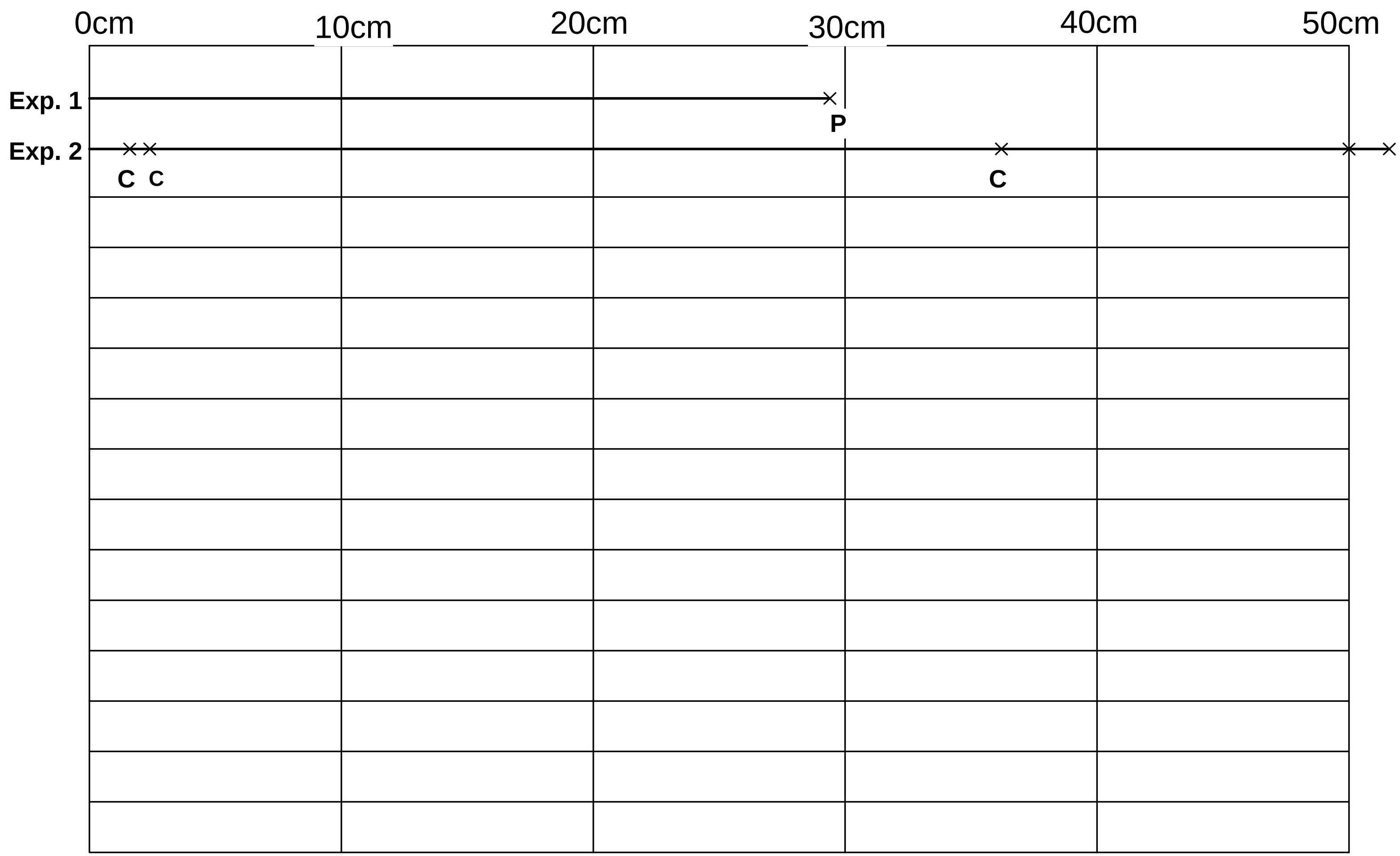
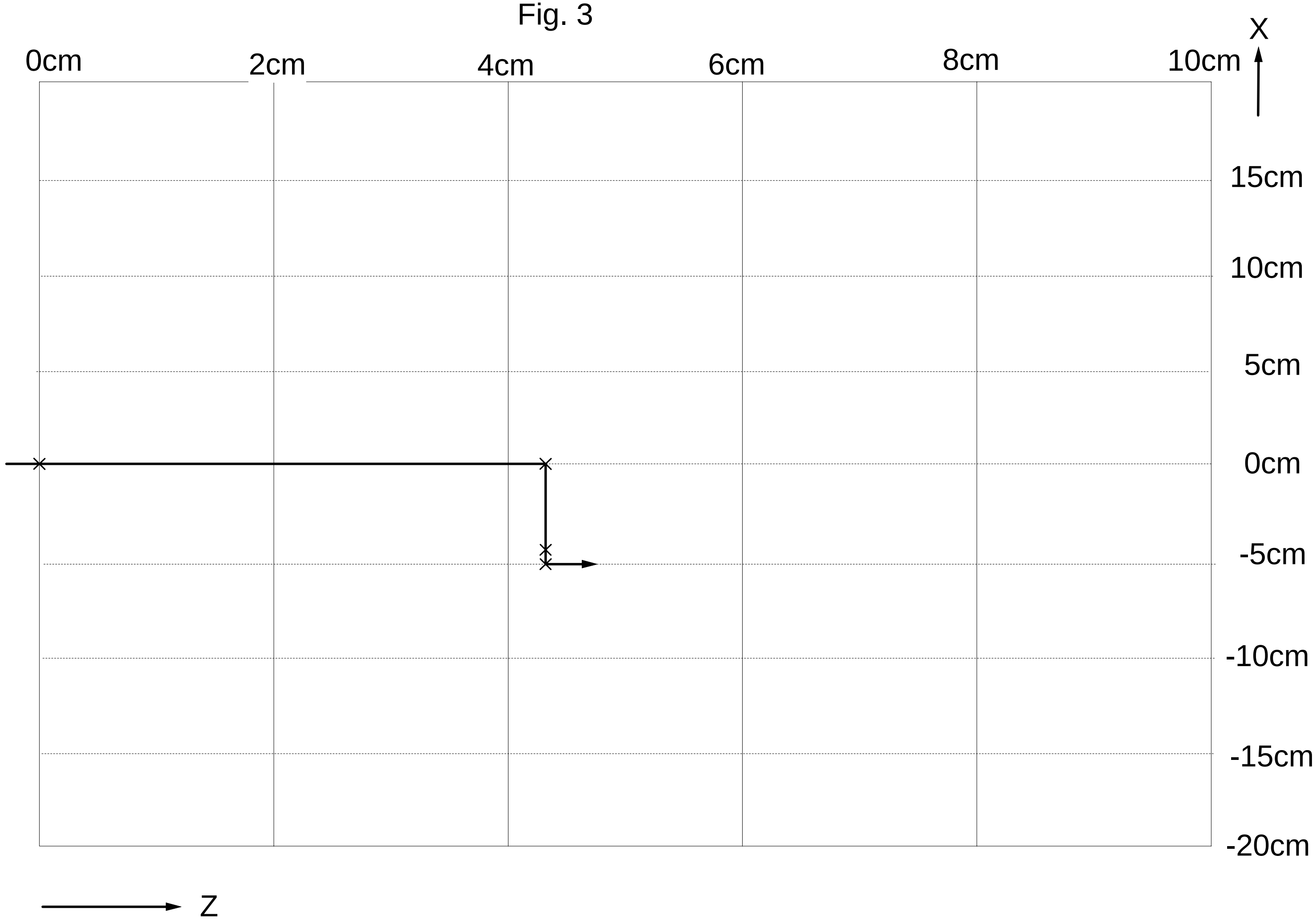


Table 3 Double Layers.

Medium A									Medium B							
No.	d(cm)	Random No.	l (cm)	d > l	d=l r d<l	Random No	Photo	Compton	d(cm)	Random No	l (cm)	d > l	d=l r d<l	Random No.	Photo	Compton
Ex. 1	40	0.32891	22.24	*		0.6116		*								
	17.76	0.2336	29.08		*				10	0.28083	3.81	*		0.906		*
									6.19	0.7162	1	*		0.9959		*
									5.19	0.6002	1.53	*		0.1871	*	

Fig. 2 Trajectories in double layers.

Fig. 3



0cm
2cm
4cm
6cm
8cm
10cm
X
15cm
10cm
5cm
0cm
-5cm
-10cm
-15cm
-20cm
Z

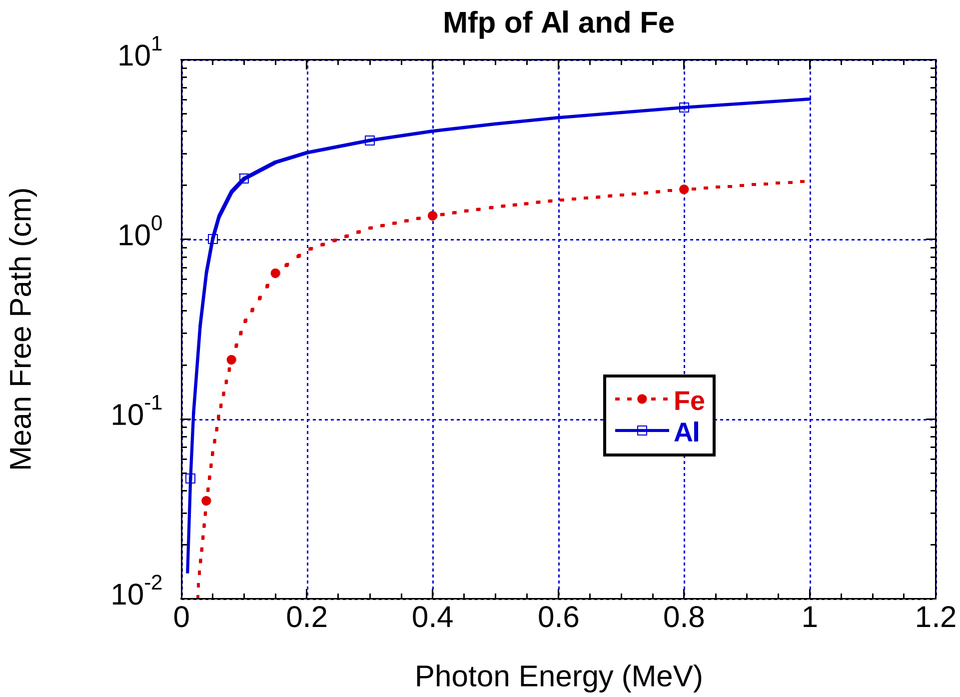


Figure 4: Mfp of Al and Fe as a function of the photon energy.

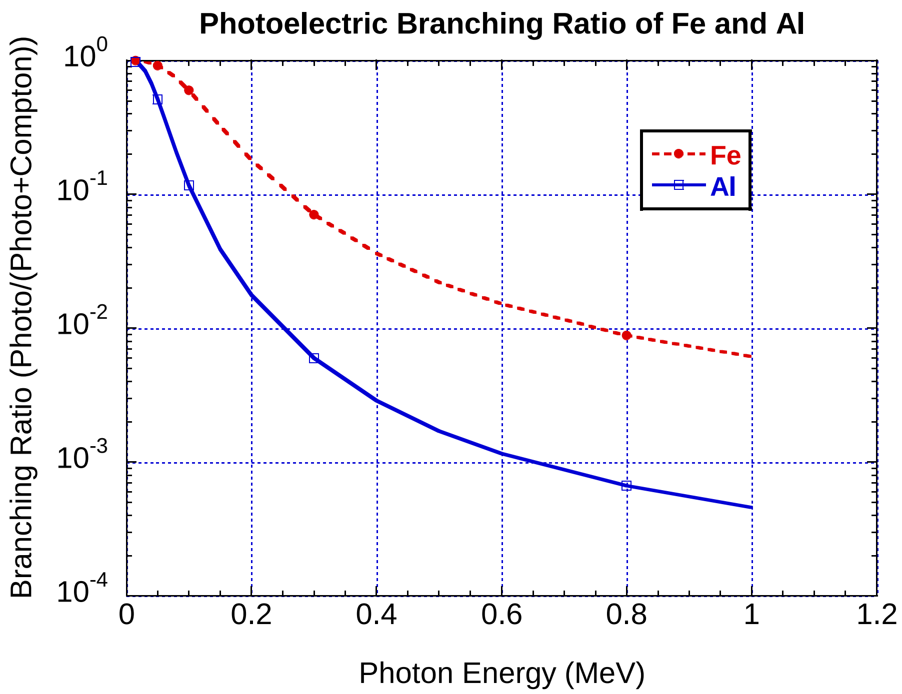
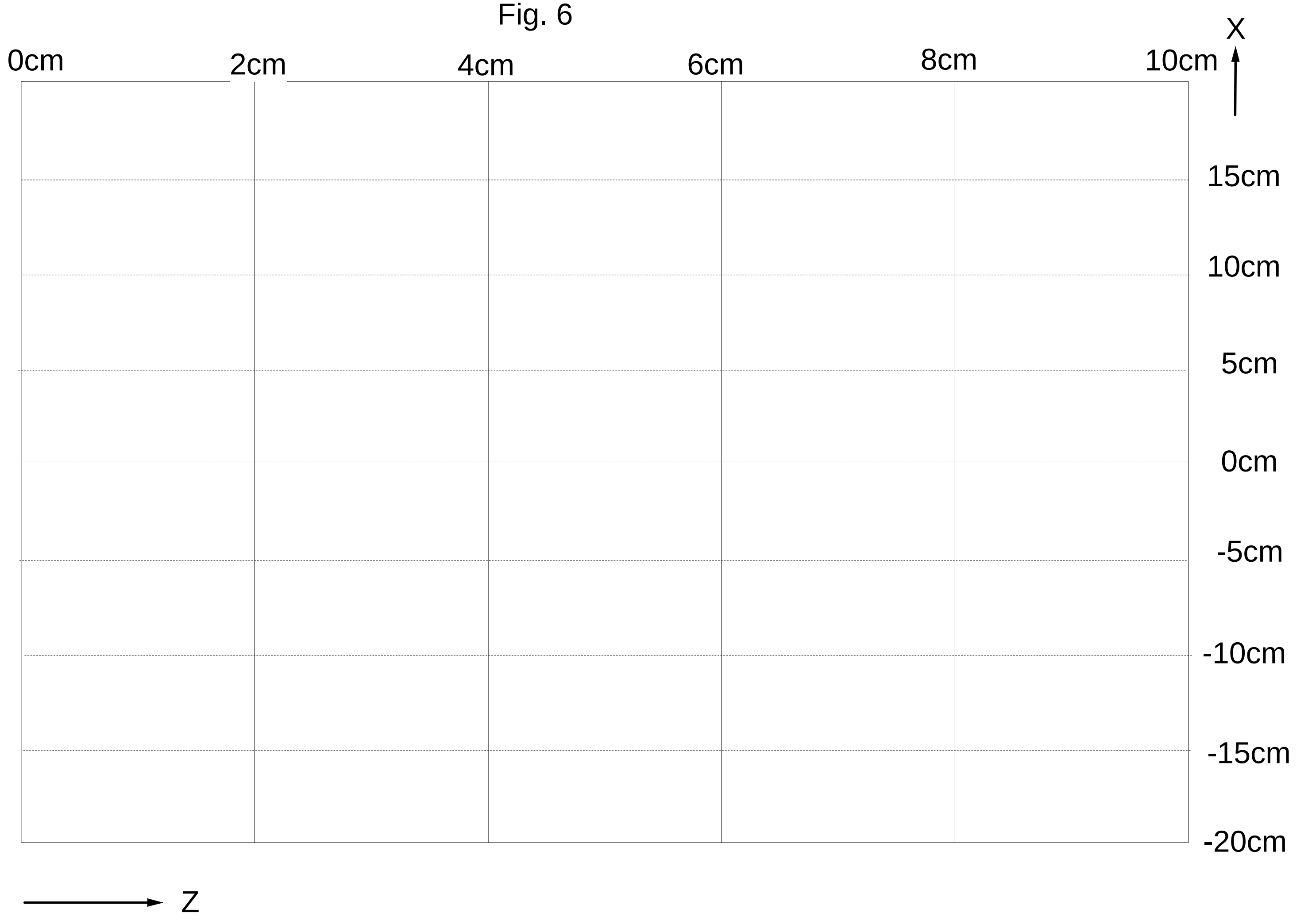


Figure 5: Photoelectric branching ratio of Al and Fe as a function of the photon energy.

Table 5

No.	E0(MeV)	Z(cm)	X(cm)	d(cm)	random number	l(cm)	d>l	d=l or d<l	randum number	Photo	Compton	random number	scattering angale	E(MeV)	random number	azimuthal angle
Source	0.5	0	0	10	0.35139	4.34	O		0.25872		O	0.57197	90 deg.	0.25	0.88784	180 deg.
Scattering 1	0.25	4.34	0	infinity	0.23809	4.79	O		0.66926		O	0.0478	0 deg.	0.25		
Scattering 2	0.25	4.34	-4.79	infinity	0.94993	0.25	O		0.32386		O	0.57888	90 deg.	0.17	0.43852	0 deg.
Scatterinb 3	0.17	4.34	-5.04	5.66												

Fig. 6


X
0cm
2cm
4cm
6cm
8cm
10cm
15cm
10cm
5cm
0cm
-5cm
-10cm
-15cm
-20cm
Z