PRINCIPAL STRESSES
AND
STRAINS

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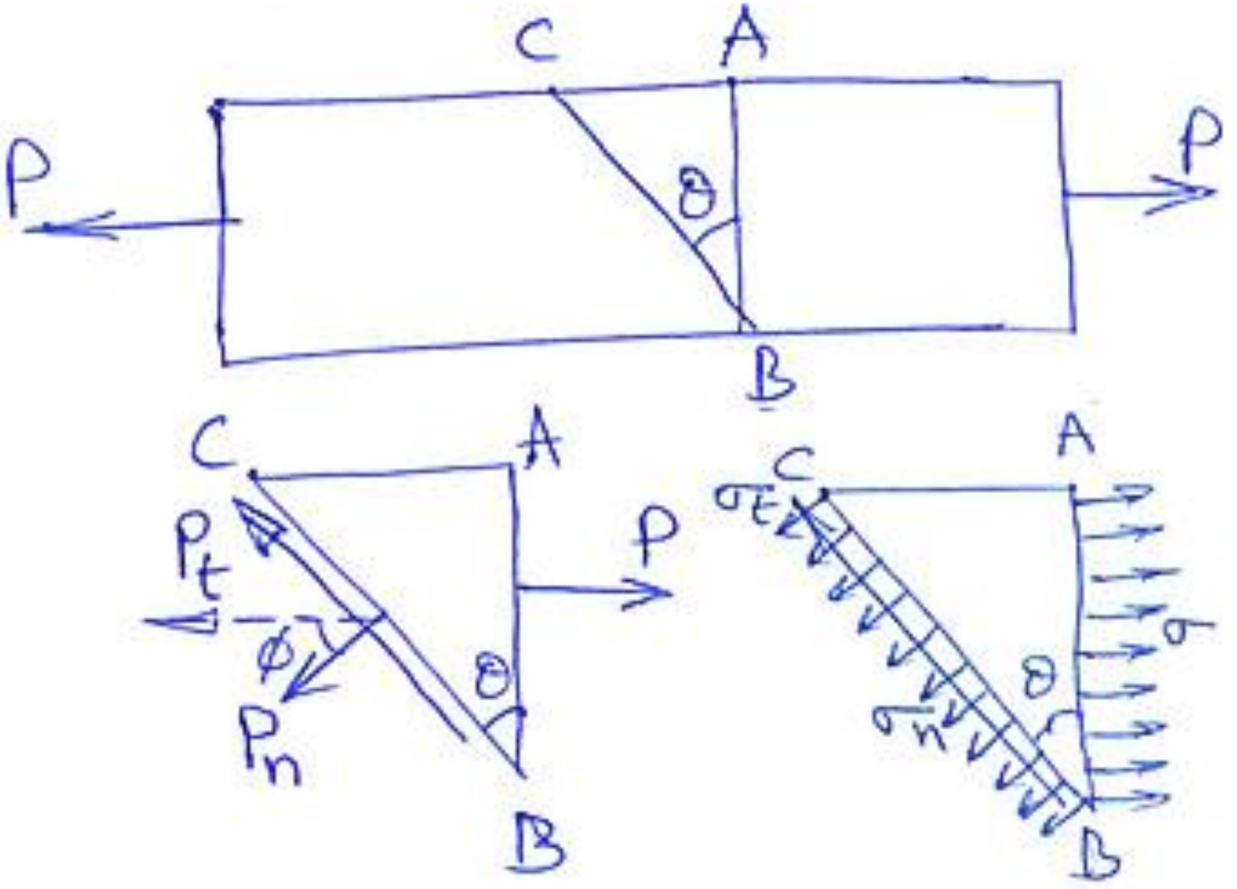
- Principal planes are these planes within the material such that the resultant stresses across them are wholly normal stresses or planes across which no shearing stresses occur.
- Principal stresses are those stresses which are acting on the principal planes.
- The plane carrying the maximum normal stress is called the major principal plane and the stress acting on it is called major principal stress.
- The plane carrying minimum normal stress is known as minor principal plane and the stress acting on it is called as minor principal stress.

Stress acting on a plane inclined to the direction of the applied force



Normal component of force, $P_n = P.\cos\theta$

Tangential component of force, $P_t = P.\sin\theta$

Normal stress, $\sigma_n = \sigma.\cos^2\theta$

Tangential stress, $\sigma_t = \sigma.\dfrac{\sin 2\theta}{2}$

P: Applied tensile force

σ: Stress

Resultant stress, $\sigma_r = \sqrt{\sigma_n^2 + \sigma_t^2}$

ϕ: Angle of the resultant stress with the normal stress

$$\tan\phi = \frac{\sigma_t}{\sigma_n}$$

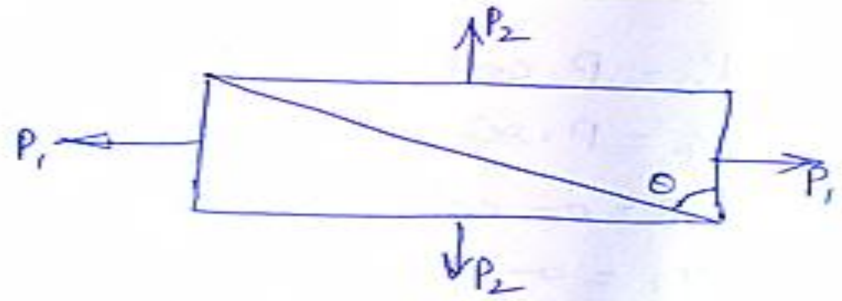
σ_n is maximum when $\cos^2\theta = 1 \Rightarrow \theta = 0^0$

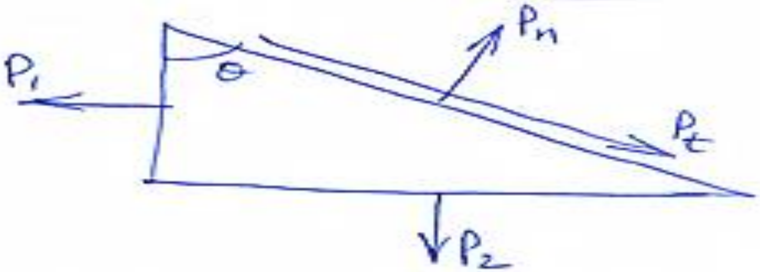
Maximum normal stress, $(\sigma_n)_{max} = \sigma$

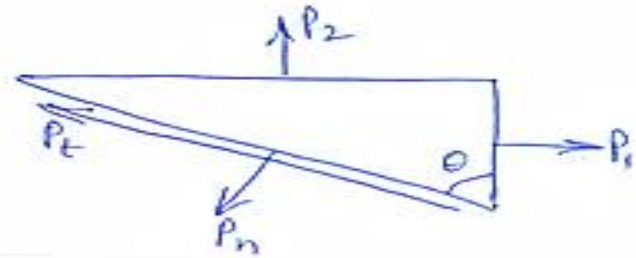
σ_t is maximum when $\sin 2\theta = \pm 1 \Rightarrow 2\theta = 90^0$ or 270^0, $\theta = 45^0$ or 135^0

Maximum tangential stress, $(\sigma_t)_{max} = \pm\dfrac{\sigma}{2}$

Stresses on an inclined plane subjected to two mutually perpendicular stresses







Normal stress, $\sigma_n = \sigma_1 . \cos^2\theta + \sigma_2 \sin^2\theta$

$$\sigma_n = \frac{\sigma_1 + \sigma_2}{2} + \frac{\sigma_1 - \sigma_2}{2} . \cos 2\theta$$

Tangential stress, $\sigma_t = \frac{\sigma_1 - \sigma_2}{2} . \sin 2\theta$

σ_t is maximum when $\sin 2\theta = \pm 1 \Rightarrow 2\theta = 90^\circ$ or 270°, $\theta = 45^\circ$ or 135°

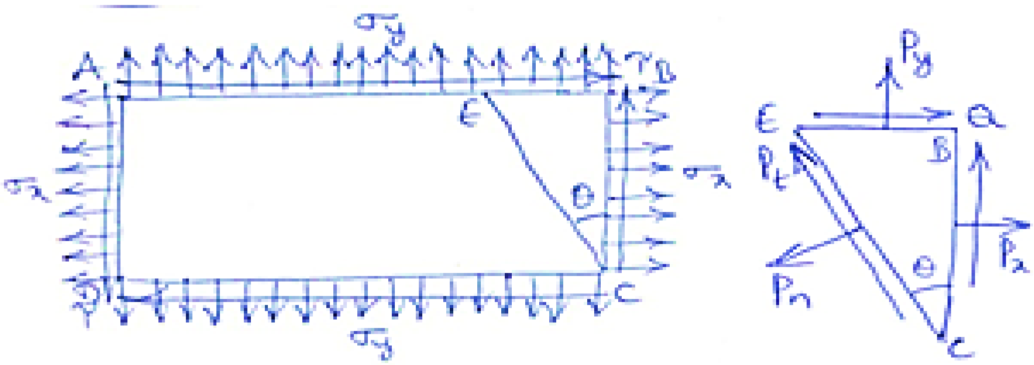
Maximum tangential stress, $(\sigma_t)_{max} = \pm \frac{\sigma_1 - \sigma_2}{2}$

Resultant stress, $\sigma_r = \sqrt{\sigma_n^2 + \sigma_t^2}$

ϕ : Angle of the resultant stress with the normal stress

$$\tan\phi = \frac{\sigma_t}{\sigma_n}$$

Stresses on an inclined plane subjected to two mutually perpendicular normal stresses and shear stress



Normal stress, $\sigma_n = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + \tau.\sin 2\theta$

Tangential stress, $\sigma_t = \dfrac{\sigma_x - \sigma_y}{2}.\sin 2\theta - \tau.\cos 2\theta$

For principal planes, $\tan 2\theta = \dfrac{2\tau}{\sigma_x - \sigma_y}$

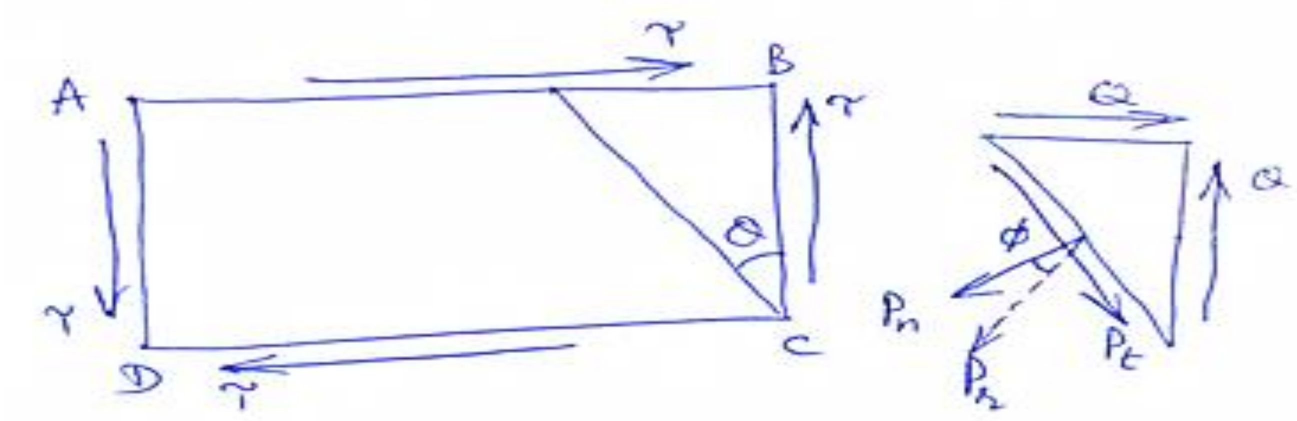
Since $\tan(180 + 2\theta) = \tan 2\theta$, there are two planes sametisfying the above relation. The value of θ differs by 90^0.

The principal stresses are: $\sigma_{1,2} = \dfrac{\sigma_x + \sigma_y}{2} \pm \dfrac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau^2}$

The principal stresses may like or unlike. The planes of maximum shear stress will be inclined at $(\theta + 45^0)$ and $(\theta + 135^0)$ to the plane BC.

Maximum shear stress, $\tau_{e,max} = \dfrac{\sigma_1 - \sigma_2}{2} = \dfrac{\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau^2}}{2}$

Stresses on an inclined plane subjected to pure shear:



Normal stress, $\sigma_n = \tau.\sin 2\theta$

Tangential stress, $\sigma_t = \tau.\cos 2\theta$

Resultant stress, $\sigma_r = \sqrt{\sigma_n^2 + \sigma_t^2}$

ϕ : Angle of the resultant stress with the normal stress

$$\tan\phi = \frac{\sigma_t}{\sigma_n} = \frac{\tau.\cos 2\theta}{\tau.\sin 2\theta} = \cot 2\theta = \tan(90 - 2\theta)$$

$$\phi = 90 - 2\theta$$

If $\sigma_t = 0$, then $\cos 2\theta = 0 \Rightarrow 2\theta = 90^\circ$ or $270^\circ \Rightarrow \theta = 45^\circ$ or 135°

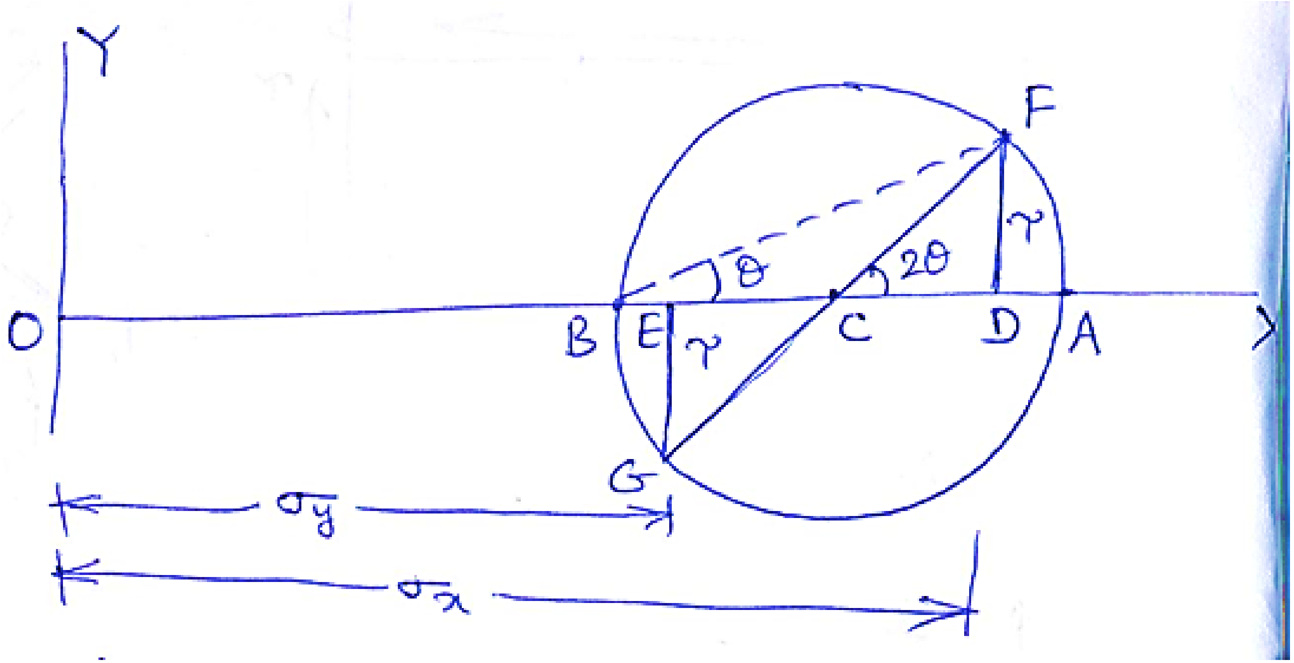
The planes inclined at 45° or 135° carriesy no tangential stress.

At $\theta = 45^\circ$, tangential stress, $(\sigma_n)_{\theta=45^\circ} = \tau$

At $\theta = 135^\circ$, tangential stress, $(\sigma_n)_{\theta=135^\circ} = -\tau$

Mohr's circle Method

1. **For finding principal stresses**



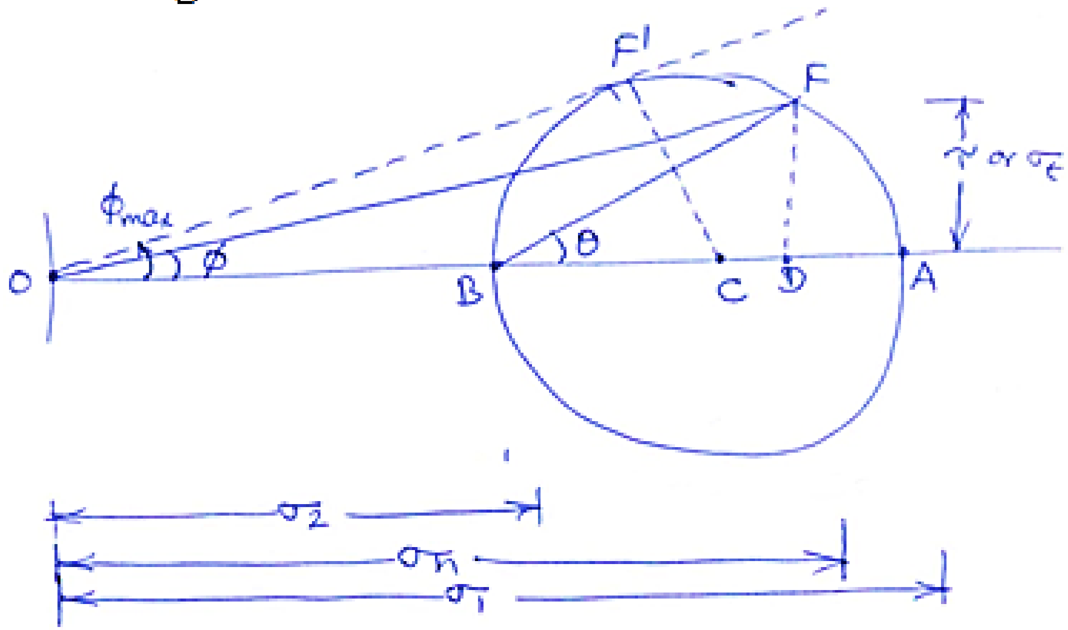
Major principal stress, $\sigma_1 = OA$

Minor principal stress, $\sigma_2 = OB$

The Radius of the circle represents the maximum shear stress.

θ : Inclination of one of the principal planes with the plane normal to which the major stress σ_x acts.

2. For finding normal stresses



Normal stress, $\sigma_n = OD$

Tangential stress, $\tau = DF$

$$\tan\phi = \frac{\sigma_t}{\sigma_n} = \frac{DF}{OD}$$

Angle of the resultant stress with the normal stress (ϕ) shall be maximum when OF is tangential to the Mohr's circle.

$$CF' = \frac{\sigma_1 - \sigma_2}{2} \; ; \; OC = \frac{\sigma_1 + \sigma_2}{2}$$

Maximum angle of the resultant stress with the normal stress (ϕ_{max}) is given by

$$\sin\phi_{max} = \frac{CF'}{OC} = \frac{\sigma_1 - \sigma_2}{\sigma_1 + \sigma_2}$$

Strains on an inclined plane:

Let ε_x : Strain in x direction

ε_y : Strain in y direction

ϕ_{xy} : Shear strain on xy plane

ε_θ : Strain on an inclined plane making an angle of θ with the major principal plane.

ϕ_θ : Shearing strain on a plane inclined at angle of θ

$$\varepsilon_\theta = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}.\cos 2\theta + \frac{\phi_{xy}}{2}.\sin 2\theta$$

$$\frac{\phi_\theta}{2} = \frac{e_x + e_y}{2}\sin 2\theta - \frac{\phi_{xy}}{2}.\cos 2\theta$$

Principal strains in two dimensional system:

$$\varepsilon_1, \varepsilon_2 = \frac{\varepsilon_x + \varepsilon_y}{2} \pm \frac{1}{2}\sqrt{(\varepsilon_x - \varepsilon_y)^2 + \phi_{xy}^2}$$

$$\tan 2\theta = \frac{\phi_{xy}}{\varepsilon_1 - \varepsilon_2}$$

Principal strains due to principal stresses:

$$\varepsilon_1 = \frac{\sigma_1}{E} - \frac{d}{m}.\frac{\sigma_2}{E} - \frac{1}{m}.\frac{\sigma_3}{E}$$

$$\varepsilon_2 = -\frac{1}{m}.\frac{\sigma_1}{E} + \frac{\sigma_2}{E} - \frac{1}{m}.\frac{\sigma_3}{E}$$

$$\varepsilon_3 = -\frac{1}{m}.\frac{\sigma_1}{E} - \frac{1}{m}.\frac{\sigma_2}{E} + \frac{\sigma_3}{E}$$

For $2D$ system,

$$\phi_{max} = \varepsilon_1 - \varepsilon_2$$

The maximum shear strain is equal to the differentce of principal strains.

Strain energy due to principal stresses:

Strain energy, $U = \frac{1}{2}.\sigma_1.\varepsilon_1 + \frac{1}{2}\sigma_2.\varepsilon_2 + \frac{1}{2}\sigma_3.\varepsilon_3$

$$= \frac{1}{2E}(\sigma_1^2 + \sigma_2^2 + \sigma_3^2) - \frac{1}{mE}(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)$$

For $2D$ system, $\sigma_3 = 0$

$$U = \frac{1}{2E}\left[\sigma_1^2 + \sigma_2^2 - \frac{2}{mE}\sigma_1\sigma_2\right]$$

Computation of principal stresses from principal strains:

$$\sigma_1 = E\left[\frac{(1-\mu)\varepsilon_1 + \mu(\varepsilon_2 + \varepsilon_3)}{(1+\mu)(1-2\mu)}\right]$$

$$\sigma_2 = E\left[\frac{(1-\mu)\varepsilon_2 + \mu(\varepsilon_3 + \varepsilon_1)}{(1+\mu)(1-2\mu)}\right]$$

$$\sigma_3 = E\left[\frac{(1-\mu)\varepsilon_3 + \mu(\varepsilon_1 + \varepsilon_2)}{(1+\mu)(1-2\mu)}\right]$$

For two dimensional stress system, $\sigma_3 = 0$

$$\sigma_1 = E.\left[\frac{\varepsilon_1 + \mu\varepsilon_2}{1-\mu^2}\right]$$

$$\sigma_2 = E.\left[\frac{\mu.\varepsilon_1 + \varepsilon_2}{1+\mu^2}\right]$$

Conditions of plane strain: $\varepsilon_z = 0,\ \phi_{yz} = 0, \phi_{zx} = 0$

Conditions of plane stress: $\sigma_z = 0, \tau_{yz} = 0, \tau_{zx} = 0$

Sum of normal strains in perpendicular directions is a constant.

$$\varepsilon_{\theta 1} + \varepsilon_{\theta 2} = \varepsilon_1 + \varepsilon_2$$

The maximum shear strain in xy plane is associated with axes at 45^0 to the direction of the principal planes.

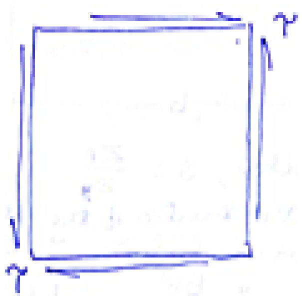
$$\frac{\phi_{max}}{2} = \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\phi_{xy}}{2}\right)^2}$$

Objective Questions and Answers in PRINCIPLE STRESSES AND STRAINS (GATE CE)

02. If an element of a stressed body is in a state of pure shear with a magnitude of 80 N/mm^2, the magnitude of maximum principal stress at that location is

a. 80 N/mm^2 b. 113.14 N/mm^2 c. 120 N/mm^2 d. 56.57 N/mm^2 CE 1999

02. a



Shear Stress, $\tau = 80\ \text{N/mm}^2$

Normal stress in x direction, $\sigma_x = 0$

Normal stress in y direction, $\sigma_y = 0$

Maximum principle stress, $\sigma_1 = \dfrac{\sigma_x + \sigma_y}{2} + \dfrac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau^2} = \tau$

$\sigma_1 = 80\ \text{N/mm}^2$

03. Pick the incorrect statement from the following four statements: CE 2000

a. On the plane which carries maximum normal stress, the shear stress is zero.

b. Principal planes are mutually orthogonal.

c. On the plane which carries maximum shear stress, the normal stress is zero

d. The principal stress axes and principal strain axes coincide for an isotropic material

03. **c**

Maximum normal stress is equal to the major principal stress. On the plane in which major principal stress acts, the shear stress is zero. Option 'a' is true.

Principal planes are mutually orthogonal . $\theta_1 = \theta_2 \pm 90^0$. Option 'b' is true

Maximum shear stress, $\tau_{max} = \dfrac{\sigma_1 - \sigma_2}{2}$

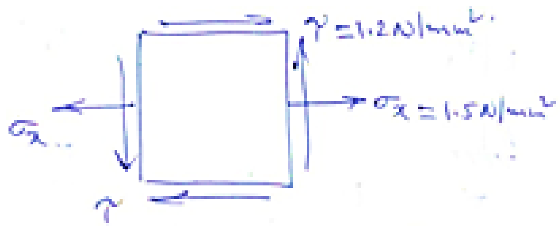
On the plane of maximum shear stress, the normal stress need not be zero. Option 'c' is false.

For an isotropic material, principal stress axes and principal strain axes coincide. Option 'd' is true.

04. The state of two dimensional stress acting on a concrete lamina consists of a direct tensile stress, $\sigma_x = 1.5$ N/mm^2, and shear stress $\tau = 1.20$ N/mm^2, which cause cracking of concrete. Then the tensile strength of the concrete in N/mm^2 is

a. 1.5 b. 2.08 c. 2.17 d. 2.29 CE 2003

04. c



Direct tensile stress, $\sigma_x = 1.5$ N/mm^2

Shear stress, $\tau = 1.20$ N/mm^2

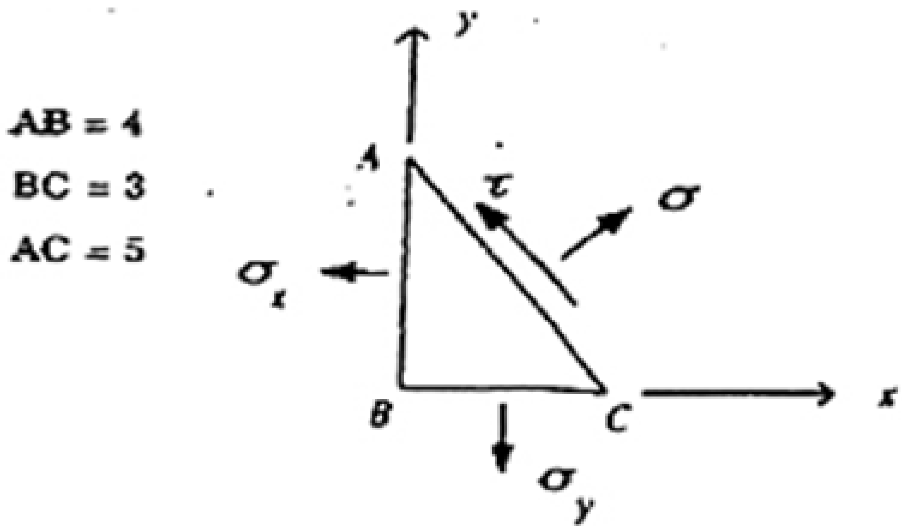
The major and minor principle stresses are given by

$$\sigma_{1,3} = \frac{\sigma_x + \sigma_y}{2} \pm \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau^2}$$

$$= \frac{1.5}{2} \pm \frac{1}{2}\sqrt{(1.5)^2 + 4(1.2)^2} = 0.75 \pm \frac{1}{2}\sqrt{2.25 + 5.76} = 0.75 \pm 1.42$$

$$\sigma_1 = 0.75 + 1.42 = 2.17 \text{ N/mm}^2$$

05. In a two dimensional stress analysis, the state of stress at a point is shown below. If σ= 120 MPa and τ = 70 MPa, then σ_x and σ_y are respectively CE 2004



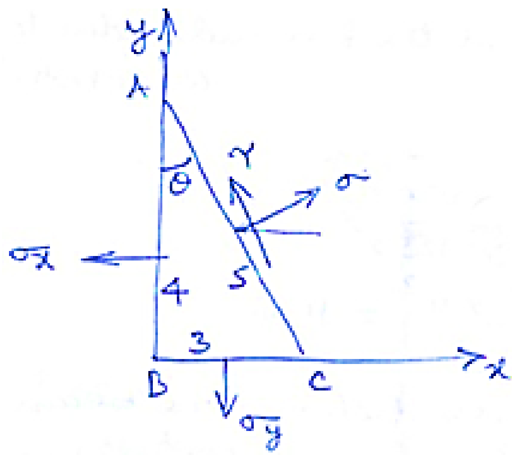
a. 26.7 MPa and 172.5 MPa

b. 54 MPa and 128 MPa

c. 67.5 MPa and 213.3 MPa

d. 16 MPa and 138 MPa

05. C



$\sigma = 120\,\text{MPa} \qquad \tau = 70\,\text{MPa}$

$\sin\theta = \frac{3}{5}, \quad \cos\theta = \frac{4}{5}, \quad \tan\theta = \frac{3}{4}$

Considering the horizontal equilibrium, $\sigma_x . AB = AC(\sigma\cos\theta - \tau\sin\theta)$

$$\sigma_x \times 4 = 5\left(120\times\frac{4}{5} - 70\times\frac{3}{5}\right) \Rightarrow \sigma_x = 67.5\,\text{MPa}$$

Considering the vertical equilibrium, $\sigma_y . BC = AC(\sigma\sin\theta + \tau\cos\theta)$

$$\sigma_y \times 3 = 5\left(120\times\frac{3}{5} + 70\times\frac{4}{5}\right) \Rightarrow \sigma_y = 213.3\,\text{MPa}$$

06. The symmetry of stress tensor at a point in the body under equilibrium is obtained from CE 2005

a. conservation of mass

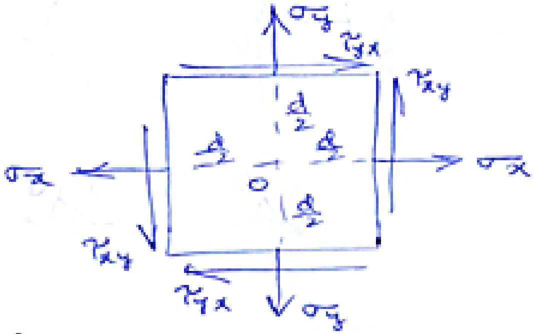
b. force equilibrium equations

c. moment equilibrium equations

d. conservation of energy

06. c

The symmetry of stress tensor at a point in the body under equilibrium is obtained from moment equilibrium equations.



Taking moments of all forces about the centre O,

$$\tau_{yx}\cdot\frac{d}{2}+\tau_{yx}\cdot\frac{d}{2}=\tau_{xy}\cdot\frac{d}{2}+\tau_{xy}\cdot\frac{d}{2}\Rightarrow\tau_{xy}=\tau_{yx}$$

08. If principal stresses in a two-dimensional case are $-10\,\text{MPa}$ and 20 MPa respectively, then maximum shear stress at the point is CE 2005

a. 10 MPa b. 15 MPa c. 20 MPa d. 30 MPa

08. b

Major Principal stress, $\sigma_1 = 20\,\text{MPa}$

Minor principal stress, $\sigma_3 = -10\,\text{MPa}$

Maximum shear stress, $\tau_{max} = \dfrac{\sigma_1 - \sigma_3}{2} = \dfrac{20-(-10)}{2} = 15\,\text{MPa}$

09. The necessary and sufficient condition for a surface to be called as a free surface is CE 2006

a. no stress should be acting on it

b. tensile stress acting on it must be zero

c. shear stress acting on it must be zero

d. no point on it should be under any stress

09. c

Free surface is the surface subjected to constant normal stress and zero tangential stress. The necessary and sufficient condition for a surface to be called as 'free surface' is shear stress acting on it must be zero.

10. Mohr's circle for the state of stress defined by $\begin{bmatrix} 30 & 0 \\ 0 & 30 \end{bmatrix}$ MPa is a circle with

a. center at (0,0) and radius 30 MPa

CE 2006

b. center at (0,0) and radius 60 MPa

c. center at (30,0) and radius 30 MPa

d. center at (30,0) and zero radius

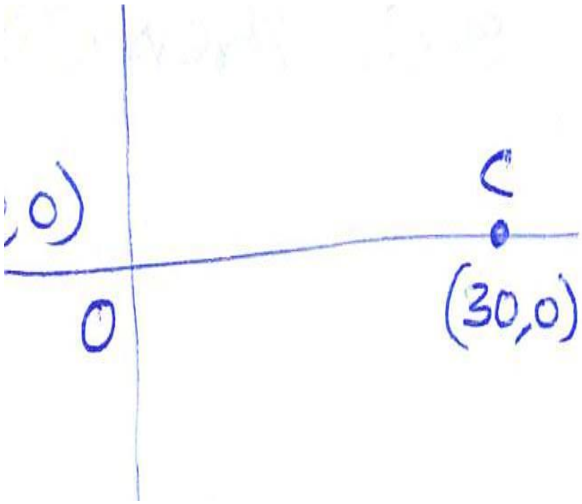
10. d

$$I = \begin{bmatrix} 30 & 0 \\ 0 & 30 \end{bmatrix}$$

$\sigma_1 = 30\,\text{MPa} \qquad \sigma_2 = 30\,\text{MPa}$

Radius of the Mohr's circle = 0

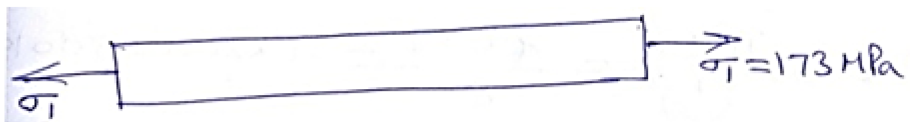
Centre of Mohr's circle: (30, 0)



11. An axially loaded bar is subjected to a normal stress of 173 MPa. The shear stress in the bar is CE 2007

a. 75 MPa b. 86.5 MPa c. 100 MPa d. 122.3 MPa

11. b



Normal stress in x direction, $\sigma_1 = 173$ MPa

Normal stress in y direction, $\sigma_2 = 0$

Maximum shear stress, $\tau_{max} = \dfrac{\sigma_1 - \sigma_2}{2} = \dfrac{173 - 0}{2} = 86.5\text{MPa}$

12. Consider the following statements CE 2009

I. On a principal plane, only normal stress acts

II. On a principal plane, both normal and shear stresses act

III. On a principal plane, only shear stress acts

IV. Isotropic state of stress is independent of frame of reference

The TRUE statements are

a. I and IV b. II c. II and IV d. II and III

12. a

Principal planes are those in which only normal stresses act and no shear stress.

Isotropic state of stress is independent of frame of reference.

13. The major and minor principal stresses at a point are 3 MPa and -3 MPa respectively. The maximum shear stress at the point is CE 2010

a. Zero b. 3 MPa c. 6 MPa d. 9 MPa

13. b

Major principal stress, $\sigma_1 = 3\,\text{MPa}$

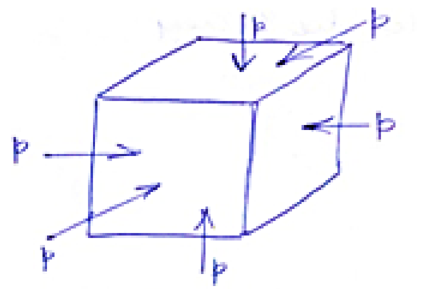
Minor principal stress, $\sigma_3 = -3\,\text{MPa}$

Maximum shear stress, $\tau_{max} = \dfrac{\sigma_1 - \sigma_3}{2} = \dfrac{3-(-3)}{2} = 3\,\text{MPa}$

14. If a small concrete cube is submerged deep in still water in such a way that the pressure exerted on all faces of the cube is p, then maximum shear stress developed inside the cube is CE 2012

a. 0 b. $\frac{p}{2}$ c. p d. $2p$

14. a



$$\sigma_x = \sigma_y = \sigma_z = p$$

Since only normal forces are acting, the shear stress $\tau = 0$

$$\tau_{max} = \frac{\sigma_1 - \sigma_2}{2} = \frac{\sigma - \sigma}{2} = 0$$

15. The state of 2D-stress at a point is given by the following matrix of stress

$$\begin{bmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{xy} & \sigma_{yy} \end{bmatrix} = \begin{bmatrix} 100 & 30 \\ 30 & 20 \end{bmatrix} \text{MPa}$$

What is the magnitude of maximum shear stress in MPa? CE 2013

a. 50 b. 75 c. 100 d. 110

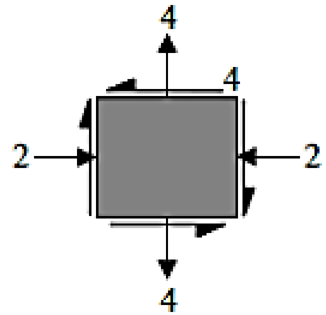
15. a

The state of 2D stress at a point is given by $\begin{bmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{xy} & \sigma_{yy} \end{bmatrix} = \begin{bmatrix} 100 & 30 \\ 30 & 20 \end{bmatrix}$ MPa

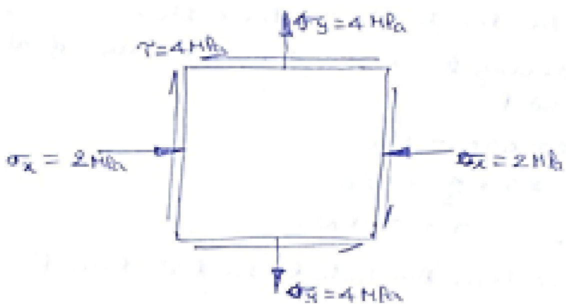
Maximum shear stress, $\tau_{max} = \pm \dfrac{\sigma_1 - \sigma_2}{2} = \dfrac{\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau^2}}{2}$

$$= \frac{1}{2}\sqrt{(100-20)^2 + 4 \times 30^2} = \frac{1}{2}\sqrt{6400 + 3600} = \frac{1}{2} \times 100 = 50\,\text{N/mm}^2$$

16. For the state of stresses (in MPa) shown in the figure below, the maximum shear stress (in MPa) is _______ CE2 2014



16. 5



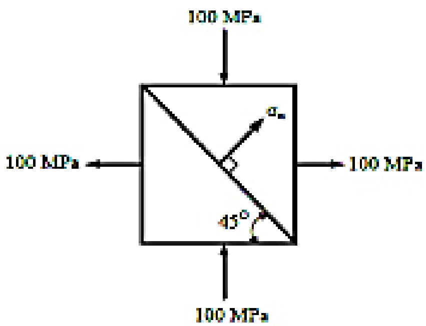
The stresses acting on an element are:

$\sigma_x = 2\,\text{MPa}\,(C), \sigma_y = 4\,\text{MPa}\,(T), \tau = 4\,\text{MPa}$

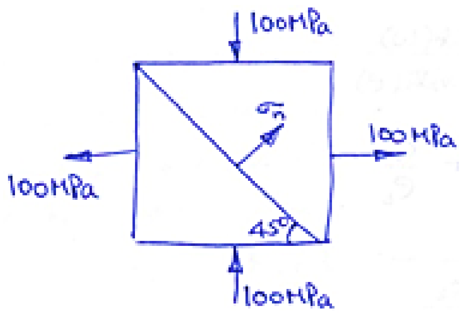
Maximum shear stress, $\tau_{max} = \dfrac{\sigma_1 - \sigma_2}{2} = \dfrac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau^2}$

$$= \frac{1}{2}\sqrt{(-2-4)^2 + 4\times 4^2} = \frac{1}{2}\sqrt{36+64} = 5\,\text{MPa}$$

17. Two triangular wedges are glued together as shown in the following figure. The stress acting normal to the interface, σ_n is...... MPa CE1 2015



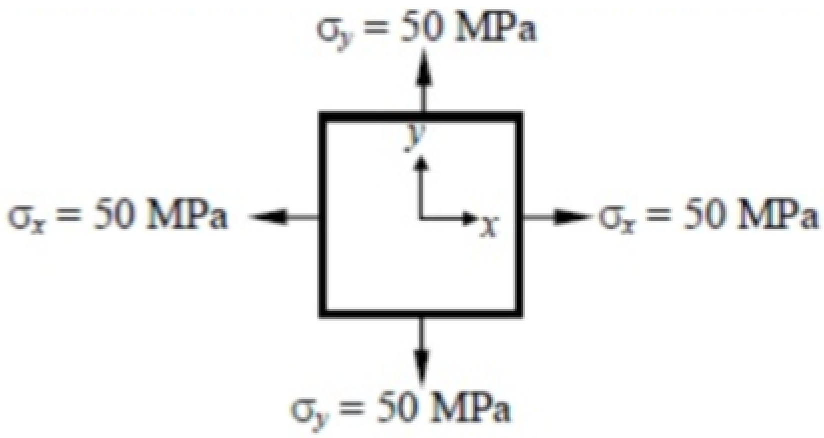
17. 0



Normal stress, $\sigma_n = \dfrac{\sigma_x + \sigma_y}{2} + \dfrac{\sigma_x - \sigma_y}{2}.\cos 2\theta$

$$= \frac{100 - 100}{2} + \frac{100 - (-100)}{2}.\cos 90^0 = 0$$

18. For the plane stress situation shown in the figure, the maximum shear stress and the plane on which it acts are: CE2 2015



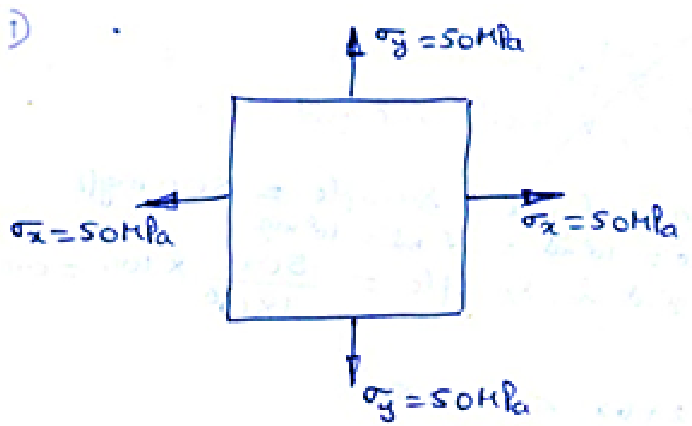
a. -50 MPa, on a plane 45^0 clockwise w.r.t x-axis

b. -50 MPa, on a plane 45^0 anti-clockwise w.r.t x-axis

c. 50 MPa, at all orientations

d. zero, at all orientations.

18. d

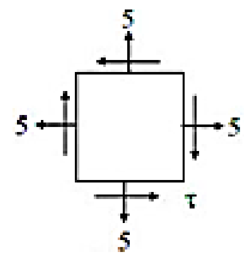


Maximum shear stress, $\tau_{max} = \dfrac{\sigma_1 - \sigma_2}{2} = \dfrac{50-50}{2} = 0$

Shear stress, $\tau_\theta = \dfrac{\sigma_x - \sigma_y}{2}.\sin 2\theta + \tau \cos 2\theta$

Since $\sigma_x - \sigma_y = 0$, $\tau_{max} = 0$ in any direction

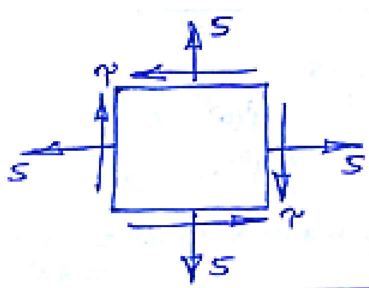
19. For the stress state in (MPa) shown in the figure, the major principal stress is 10 MPa. CE2 2016



The shear stress τ is

a. 10.0 MPa **b.** 5.0 MPa c. 2.5 MPa d. 0.0 MPa

19. b



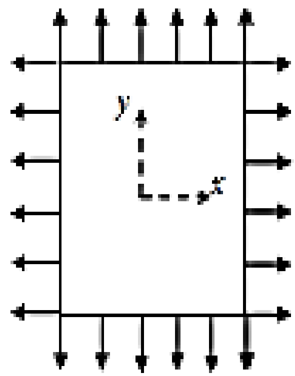
Major principal stress, σ_1=10 MPa

$\sigma_x = 5$ MPa, $\sigma_y = 5$ MPa

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau^2} = \frac{5+5}{2} + \frac{1}{2}\sqrt{0+4\tau^2} = 5 + \tau$$

$10 = 5 + \tau \Rightarrow \tau = 5$ MPa

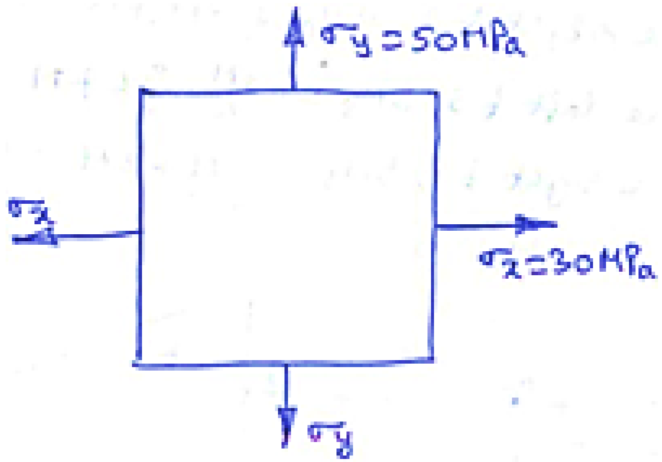
21. A plate in equilibrium is subjected to uniform stresses along its edges with magnitude σ_{xx} =30 MPa and σ_{yy} =50 MPa as shown in the figure. CE1 2018



The Young's modulus of the material is 2×10^{11} N/m^2 and the Poisson's ratio is 0.3. If σ_{zz} is negligibly small and assumed to be zero, then the strain ε_{zz} is

a. -120×10^{-6} b. -60×10^{-6} c. 0.0 d. 120×10^{-6}

21. a



Young's modulus of material, $E = 2\times10^{11}$ N/m^2

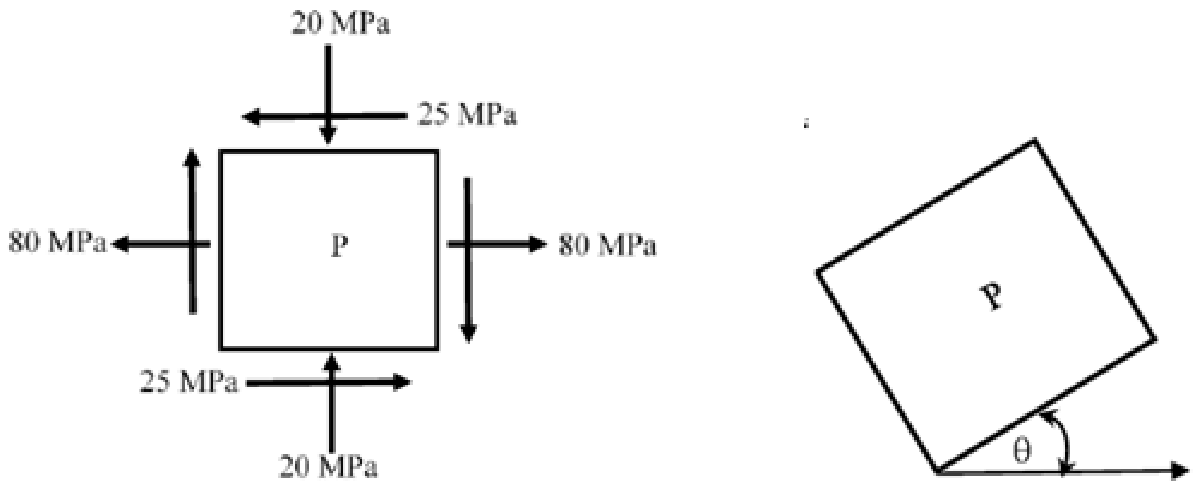
Poisson's ratio, $\mu = 0.3$

$\sigma_x = 30$ Mpa, $\sigma_y = 50$ Mpa and $\sigma_z = 0$

Strain in z direction, $\varepsilon_z = -\mu.\dfrac{\sigma_x}{E} - \mu.\dfrac{\sigma_y}{E} + \dfrac{\sigma_z}{E}$

$$\varepsilon_z = \frac{-\mu(\sigma_x + \sigma_y) + \sigma_z}{E} = \frac{-0.3(30+50)+0}{2\times10^{11}\times10^{-6}} = -1.2\times10^{-4} = -120\times10^{-6}$$

23.For a plane stress problem, the state of stress at a point P is represented by the stress element as shown in figure.



By how much angle (θ) in degrees the stress element should be rotated in order to get the planes of maximum shear stress? CE2 2019

a. 13.3 b. 26.6 **c.** 31.7 d. 48.3

23. C

Given $\sigma_x = 80$ MPa, $\sigma_y = -20$ MPa, , $\tau = 80$ MPa,

Angle of major principal plane is given by

$$\tan 2\theta \quad \tan 2\theta = \frac{2\tau}{\sigma_x - \sigma_y} = \frac{2(-25)}{80-(-20)} = -0.5$$

$$2\theta = -26.56^0 \Rightarrow \theta = -13.28^0$$

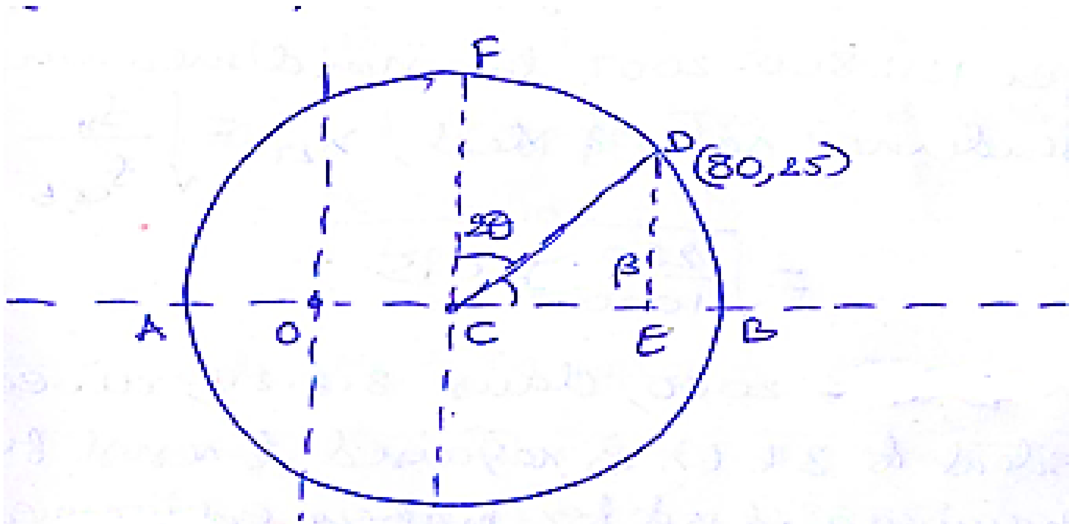
Angle of plane of maximum shear stress, $\alpha = \theta + 45^0$

$$\alpha = -13.28^0 + 45 = 31.72^0$$

Or

Mohr circle method,

Given $\sigma_x = 80$ MPa, $\sigma_y = -20$ MPa, , $\tau = 80$ MPa,



$$AC = \frac{80+20}{2} = 50 \text{ MPa}$$

Centre of Mohr circle: $C\left(\frac{\sigma_x + \sigma_y}{2}, 0\right) = C\left(\frac{80-20}{2}, 0\right)$ =C(30,0)

Radius of Mohr circle, $R = \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau^2}$

$$R = \frac{1}{2}\sqrt{(80-(-20))^2 + 4\times 25^2} = 55.9 \text{ MPa}$$